

On even-closedness of vertex-transitive graphs

Dragan Marušič

University of Primorska, Koper, Slovenia

dragan.marusic@upr.si

When dealing with symmetry properties of combinatorial objects, such as graphs admitting a transitive group action, that is, *vertex-transitive graphs*, one of the fundamental questions is to determine their full automorphism group. While some symmetries of such objects are obvious, certain additional symmetries remain hidden or difficult to grasp. When this is the case, the goal is to find a reason for their existence and a method for describing them.

Along these lines the above question reads as follows: *Given a transitive group H acting on a set V of vertices of a graph, determine whether H is its full automorphism group or not. When the answer is no, find a method to describe the additional automorphisms.*

We propose to study such group “extensions” by considering the existence of *odd automorphisms* (as opposed to even automorphisms) that is automorphisms that act as odd permutations on the vertex set of a graph. The implications go beyond the simplicity of the concept of even/odd permutations alone. For example, given a group H consists of even permutations only, a partial answer to the above question could be given provided the structure of the graph in question forces existence of automorphisms acting as odd automorphisms.

In this talk some recent results in regards to the above problem will be considered. A special emphasis will be given to the class of cubic symmetric graphs where a complete solution will be presented. These results suggest that the even/odd question is likely to uncover certain much more complex structural properties of graphs that go beyond simple arithmetic conditions.