New infinite family of Cameron-Liebler line classes

Ilya Matkin
Chelyabinsk State University, Chelyabinsk, Russia
ilya.matkin@gmail.com

This is joint work with Alexander Gavrilyuk and Tim Penttila

Let $\mathrm{PG}(3, q)$ denote the 3-dimensional projective space over the finite field $\mathbb{F}_q$. A *Cameron-Liebler line class* of $\mathrm{PG}(3, q)$ is a set of lines that shares a constant number x of lines with every spread of $\mathrm{PG}(3, q)$. The number x is called the *parameter* of the Cameron-Liebler line class. It can be seen from the definition that the complementary set to a Cameron-Liebler line class is a Cameron-Liebler line class with parameter $q^2 + 1 - x$, so that we may assume $x \leq (q^2 + 1)/2$.

The examples of Cameron-Liebler line classes include an empty set of lines ($x = 0$), the set of all lines in a plane ($x = 1$) or, dually, through a point ($x = 1$), and, the union of the previous two examples with $x = 1$, assuming that the point is not in the plane ($x = 2$). Cameron-Liebler line classes first appeared in the study [1] (see also [8]) on collineation groups of $\mathrm{PG}(n, q)$, $n \geq 3$, that have equally many orbits on lines and on points. For more comprehensive background, we refer to recent papers [3–6].

It was conjectured in [1] that the only Cameron-Liebler line classes are the examples mentioned above, i.e., $x \leq 2$. The first counterexample was found by Drudge in $\mathrm{PG}(3, 3)$ with $x = 5$, which was generalised later by Bruen and Drudge [2] to an infinite family having parameter $x = (q^2 + 1)/2$ for all odd q. With the aid of computer Rodgers [7] constructed many more new examples for certain x and prime powers q. Some of them have been shown in [3], [6] to be a part of a new infinite family of Cameron-Liebler line classes with parameter $x = (q^2 + 1)/2$ for $q \equiv 5$ or $9 \pmod{12}$.

In this work, we construct one more infinite family of Cameron-Liebler line classes in $\mathrm{PG}(3, q)$ with parameter $x = (q^2 + 1)/2$ for all odd q, which are somehow related to the line classes of Bruen and Drudge, but not equivalent to them. In particular, for $q = 5$, there exist at least 3 pairwise non-equivalent Cameron-Liebler line classes with $x = (q^2 + 1)/2 = 13$.

References

[1] P. J. Cameron, R. A. Liebler, Tactical decompositions and orbits of projective groups. *Linear Algebra Appl.* **46** (1982) 91–102.

[2] A. A. Bruen, K. Drudge, The construction of Cameron-Liebler line classes in $\mathrm{PG}(3, q)$. *Finite Fields Appl.* **5(1)** (1999) 35–45.

[3] T. Feng, K. Momihara, Q. Xiang, Cameron-Liebler line classes with parameter $x = \frac{q^2-1}{2}$. *J. Combin. Theory Ser. A* **133** (2015) 307-338.

[4] A. L. Gavrilyuk, K. Metsch, A modular equality for Cameron-Lieber line classes. *J. Combin. Theory Ser. A* **127** (2014) 224–242.

[5] A. L. Gavrilyuk, I. Yu. Mogilnykh, Cameron-Liebler line classes in $\mathrm{PG}(n, 4)$. *Des. Codes Cryptogr.* **73(3)** (2014) 969–982.

[6] K. Metsch, J. De Beule, J. Demeyer, M. Rodgers, A new family of tight sets in $Q^+(5, q)$. *Des. Codes Cryptogr.* **78** (2016) 655–678.

[7] M. Rodgers, Cameron-Liebler line classes. *Des. Codes Cryptogr.* **68(1-3)** (2013) 33–37.

[8] T. Penttila, Cameron-Liebler line classes in $\mathrm{PG}(3, q)$. *Geom. Dedicata* **37(3)** (1991) 245–252.