

A q and q, t -analogue of Hook Immanantal Inequalities and Hadamard Inequality for Trees

Mukesh Kumar Nagar

Department of Mathematics, IIT Bombay, India

mukesh.phd@math.iitb.ac.in

Sivaramakrishnan Sivasubramanian

Department of Mathematics, IIT Bombay, India

krishnan@math.iitb.ac.in

Let T be a tree on n vertices with Laplacian matrix L and q -Laplacian \mathcal{L}_q . Let χ_k be the character of the irreducible representation of the symmetric group \mathfrak{S}_n indexed by the hook partition $k, 1^{n-k}$ and let $\bar{d}_k(L)$ be the normalized hook immanant of L corresponding to the character χ_k . In [1, 3–5], inequalities are known for $\bar{d}_k(L)$ as k increases. By using matchings and assigning statistics to vertex orientations, we generalize these inequalities to the matrix \mathcal{L}_q , for all $q \in \mathbb{R}$ and to the bivariate q, t -Laplacian $\mathcal{L}_{q,t}$ for a specific set of values q, t , where both $q, t \in \mathbb{R}$ or both $q, t \in \mathbb{C}$. Our statistic based approach also gives generalizations of inequalities given in [2] for a Hadamard inequality changing index $k(L)$ of L , to the matrices \mathcal{L}_q and $\mathcal{L}_{q,t}$ for trees.

References

- [1] O. Chan, T. K. Lam, Hook immanantal inequalities for trees explained. *Linear Algebra and its Applications* **273** (1998) 119–131.
- [2] O. Chan, B. Ng, Hook immanantal inequalities for hadamard's function. *Linear Algebra and its Applications* **299** (1999) 175–190.
- [3] P. Heyfron, Immanant dominance orderings for hook partitions. *Linear and Multilinear Algebra* **24** (1988) 65–78.
- [4] E. H. Lieb, Proof of some conjectures on permanents. *Journal of Math and Mech.* **16** (1966) 127–134.
- [5] I. Schur, Über endliche gruppen und hermitesche formen. *Math. Z.* **1** (1918) 184–207.