Classifying the fuzzy subgroups of finite symmetric groups

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The main goal of this paper is to classify the fuzzy subgroups of the finite symmetric group $S_n$. First, an equivalence relation on the set of all fuzzy subgroups of a group $G$ is defined. Without any equivalence relation on fuzzy subgroups of group $G$, the number of fuzzy subgroups is infinite, even for the trivial group. In this paper, classifying the fuzzy subgroups structure of a finite symmetric groups $S_n (n \geq 5)$ is made. An explicit formula for the number of distinct fuzzy subgroups of $S_n$ is indicated. We also count the number of fuzzy subgroups for a particular class of finite symmetric groups.

One of the most important problems of fuzzy group theory is to classify the fuzzy subgroups of a finite groups. This topic has enjoyed a rapid development in the last few years. In our case the corresponding equivalence classes of fuzzy subgroups are closely connected to the chains of subgroups in $S_n$. As a guiding principle in determining the number of these classes, we found the number of maximal chains of $S_n$. Note that an essential role in solving our counting problem is played again by the Inclusion-Exclusion Principle. It leads us to some recurrence relations, whose solutions have been easily found.