On Characterizations of Association Schemes by Intersection Numbers

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A coherent configuration $\mathcal{X}$ is characterized by the intersection numbers if every algebraic isomorphism of this configuration to another one is induced by a combinatorial isomorphism; in this case, $\mathcal{X}$ is said to be separable. The importance of this notion is explained by the fact that if the coherent configuration of a graph is separable, then the isomorphism of this graph to any other graph can be tested by the Weisfeiler-Leman algorithm [4]. Besides, the separability of a distance-regular graph (or, more general, of an association scheme) means in terms of [1], that the graph is uniquely determined by its parameters.

The index of an association scheme $\mathcal{X}$ with n points and m relations of valency 1 is defined to be the number n/m. In [2, 3], it was proved that every quasi-thin or pseudocyclic scheme $\mathcal{X}$ is separable whenever the index of $\mathcal{X}$ is enough large in comparison with its maximal valency. It turns out that a similar result (with much better bound) holds for the class of TI-schemes, which contains the most part of quasi-thin schemes and all pseudocyclic schemes. Here, a TI-scheme can be thought as a combinatorial analog of the coherent configuration of a transitive group G, the point stabilizer of which is a TI-subgroup of G.

As a byproduct of the main result, we prove that every association scheme of prime degree p and valency k is schurian, whenever $p > 1 + 6k(k-1)^2$. This improves [3, Corollary 1.2], where the lower bound for p was $O(k^5)$.

References

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