On the isomorphism problem for Cayley graphs over abelian p-groups

Grigory Ryabov
Novosibirsk State University, Novosibirsk, Russia
gric2ryabov@gmail.com

Let G be a finite group. A Schur ring over G is a subring of the group ring $\mathbb{Z}G$ that has a linear basis associated with a special partition of G. About 40 years ago Pöschel described all S-rings over cyclic p-groups of odd order. Applying this result Pöschel and Klin solved the isomorphism problem for circulant graphs with p^k vertices, where p is an odd prime.

Let $n = p^{k+1}$, where $p \in \{2, 3\}$ and k is a positive integer. Denote by $\mathcal{G}_n$ and $\mathcal{P}_n$ the class of all graphs on n vertices and the class of graphs on n vertices that isomorphic to Cayley graphs over $G = \mathbb{Z}_p \times \mathbb{Z}_{p^k}$ respectively. Recently all S-rings over G were classified in [1] for $p = 2$ and in [2] for $p = 3$. By using this classification we prove the following theorem.

Theorem. *In the above notation, suppose that the group G is given by its multiplication table. Then the following problems can be solved in time $n^{O(1)}$:*

(1) *given a graph $\Gamma \in \mathcal{G}_n$, test whether $\Gamma \in \mathcal{P}_n$;*

(2) *given graphs $\Gamma, \Gamma' \in \mathcal{P}_n$, test whether $\Gamma \cong \Gamma'$, and (if so) find the set of all isomorphisms between them.*

References

[1] M. Muzychuk, I. Ponomarenko, On Schur 2-groups, Zapiski Nauchnykh Seminarov POMI. **435** (2015) 113–162.

[2] G. Ryabov, On Schur p-groups of odd order. *J. Algebra Appl.* (2016) DOI: 10.1142/S0219498817500451.