On the isomorphism problem for Cayley graphs over abelian $p$-groups

Grigory Ryabov
Novosibirsk State University, Novosibirsk, Russia
gric2ryabov@gmail.com

Let $G$ be a finite group. A Schur ring over $G$ is a subring of the group ring $\mathbb{Z}G$ that has a linear basis associated with a special partition of $G$. About 40 years ago Pöschel described all $S$-rings over cyclic $p$-groups of odd order. Applying this result Pöschel and Klin solved the isomorphism problem for circulant graphs with $p^k$ vertices, where $p$ is an odd prime.

Let $n = p^{k+1}$, where $p \in \{2, 3\}$ and $k$ is a positive integer. Denote by $G_n$ and $P_n$ the class of all graphs on $n$ vertices and the class of graphs on $n$ vertices that isomorphic to Cayley graphs over $G = \mathbb{Z}_p \times \mathbb{Z}_{p^k}$ respectively. Recently all $S$-rings over $G$ were classified in [1] for $p = 2$ and in [2] for $p = 3$. By using this classification we prove the following theorem.

Theorem. In the above notation, suppose that the group $G$ is given by its multiplication table. Then the following problems can be solved in time $n^{O(1)}$:

1. given a graph $\Gamma \in G_n$, test whether $\Gamma \in P_n$;
2. given graphs $\Gamma, \Gamma' \in P_n$, test whether $\Gamma \cong \Gamma'$, and (if so) find the set of all isomorphisms between them.

References
