# Homotopy type of neighborhood complexes of Kneser graphs

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A. Schrijver identified a family of vertex critical subgraphs of Kneser graphs called the stable Kneser graphs $SG_{n,k}$. A. Bjórner and M. de Longueville proved that the neighborhood complex of the stable Kneser graph $SG_{n,k}$ is homotopy equivalent to a k-sphere. It is also known that the neighborhood complex of $KG_{n,k}$ is homotopy equivalent to the wedge sum of k-spheres. The main objective here is to give the exact number for $KG_{2,k}$ i.e. to show that the homotopy type of the neighborhood complex of $KG_{2,k}$ is a wedge sum of $(k+4)(k+1)+1$ spheres of dimension k. Further we will construct a subgraph $S_{2,k}$ of $KG_{2,k}$ whose neighborhood complex deformation retracts onto the neighborhood complex of $SG_{2,k}$.

## References

[1] L. Lovász, Kneser's conjecture, chromatic number, and homotopy. *J. Combin. Theory Ser. A* **25(3)** (1978) 319–324.

[2] A. Schrijver, Vertex critical subgraphs of Kneser graphs. *Nieuw Arch. Wisk. (3)* **26(3)** (1978) 454–461.

[3] A. Björner, M. de Longueville, Neighborhood complexes of stable Kneser graphs. *Combinatorica* **23(1)** (2003) 23–34.

[4] D. Kozlov, *Combinatorial algebraic topology.* Vol. 21. Algorithhms and Computation in Mathematics, Springer, Berlin, 2008.

[5] B. Braun, M. Zeckner, Deformation ratracts of neighborhood complexes of stable Kneser graphs. *Proceedings of the American Mathematical society* **142(2)** (2014) 413-427.

[6] B. Braun, Independence complexes of stable Kneser graphs. *Electronic Journal of Combinatorics, 18* **1** (2011) 118.

[7] A. Dochtermann, C. Schultz, Topology of Hom complexes and test graphs for bounding chromatic number. *Israel Journal of Mathematics*, **187(1)** (2012) 371-417.

[8] J. Jonsson, *Simplicial complexes of graphs.* Vol. 1928 of Lecture notes in Mathematics, Springer-Verlag, Berlin, 2008.

[9] R. Forman, Morse theory for cell complexes. *Adv. Math.* **134** (1998) 90-145.