Minimal generating systems and structure of Sylow 2-subgroups of alternating groups $Syl_2A_{2^k}$ and Syl_2A_n

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The aim of this investigation is to research the structure of Sylow 2-subgroups of A_n and to construct a minimal generating system for it. The authors of the article [1] didn't prove minimality of the system of generators for the Sylow 2-subgroups of A_n, which was found by them, and its was not fully investigated in [1,2]. The case of Sylow 2-subgroups is very special, because group $C_2 \wr C_2 \wr C_2 \ldots \wr C_2$ can acts by odd permutations on X^k [2], where $X = \{0,1\}$. Let us denote by $X^{[k]}$ a regular binary rooted tree with k levels.

There was a mistake in a statement about irreducibility of the generating system of $k+1$ elements for $Syl_2(A_{2^k})$, which was in abstract [3] in year 2015.

Theorem 1. *A maximal 2-subgroups G_k of $AutX^{[k]}$ that acts by even permutations on X^k have structure of inner semidirect product $G_k \simeq \underbrace{(S_2 \wr S_2 ... \wr S_2)}_{k-1} \ltimes \underbrace{(C_2 \times ... \times C_2)}_{2^{k-1}-1}$ and isomorphic to $Syl_2A_{2^k}$.*

In accordance with Legendre formula, the power of 2 in $2^k!$ is $\frac{2^k-1}{2-1}$. We need to subtract 1 from 2^k-1, because we have only $\frac{2^k!}{2}$ of permutations. $|G_k| = |\underbrace{S_2 \wr ... \wr S_2}_{k-1}| \cdot |\underbrace{C_2 \times ... \times C_2}_{2^{k-1}-1}| = |Syl_2A_{2^k}| = 2^{2^k-2}$.

Statement 1. *A quotient group ${}^{G_k}/_{G_k^2G_k'}$ is isomorphic to $\underbrace{C_2 \times C_2 \times ... \times C_2}_{k}$.*

Proof follows from that $G_k^2G_k' \simeq G_k^2 \triangleleft G_k$ and $\left|G : G_k^2G_k'\right| = 2^k$. Also it was deduced that derived length of $Syl_2A_2^k$ is not always equal to k, because in case $k=2$ the Sylow subgroup of A_{2^k} is $Syl_2A_4 \simeq K_4$ but K_4 is abelian group, so its derived length is 1. But it was said in Lemma 3 of [1] that derived length of $Syl_2A_2^k$ is always equal to k if $k \geq 1$, so it is a mistake.

Main Theorem. *A minimal generating system of Sylow 2-subgroups of alternating group A_{2^k} consists of k elements.*

There is Morse function [4] $f : D^2 \to \mathbb{R}$ which has 2^k points of maximum and a graph of Kronrod-Reeb [5], which is correspondent to it and is obtained by contraction of every set's component of level $f^{-1}(c)$ in a point. The group of automorphism of this graph is isomorphic to $Syl_2S_{2^k}$.

References

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