27/8/2016

SPECTRAL GRAPH THEORY EXERCISES SET #2

1. Prove that $\chi_{vec}(G) \le \chi(G)$. [easy]
2. Determine $\chi_{vec}(C_5)$ or find good lower/upper bounds. What does "Hoffman" say?
3. Prove that every tree of maximum degree d has all eigenvalues in $[-2\sqrt{d-1}, 2\sqrt{d-1}]$. (Try proving $(-2\sqrt{d-1}, 2\sqrt{d-1})$.)
   Hint: Remember T_d and $T_{d,r}$.

4. Star-like tree is a tree with only one vertex of degree ≥ 3. Prove that all eigenvalues except possibly λ_1 and λ_n of any starlike tree lie strictly between -2 and 2.
   Hint. Use interlacing.

5. Determine all eigenvalues of the Heawood graph.
   Hint: remember a pf from the lectures. using BB^T.
6. Draw the Heawood graph on the torus (dual of K_7).

7. Group project (8 students): Find characteristic polynomials of all orientations of K_4. There are 2^6 orientations but because of "switching" you may assume , so there are 8 cases.
   Discuss their eigenvalues.

8. View G as an electrical network, edges having resistance 1. Let r_{uv} be the resistance from u to v.
   Compute $\sum_{u \in V} \sum_{v \in V} r_{uv} = \sum_{\lambda \in \sigma_L(G) \setminus \{0\}} \frac{1}{\lambda}$ (sum of inverse non-zero eig's of the Laplacian)
   for random cubic graphs using McKay-Kesten formula.