On plateaued Boolean functions with the same spectrum support

Yuriy Tarannikov
Moscow State University, Moscow, Russia
yutarann@gmail.com

The Boolean function f is a map $\mathbf{F}_2^n \to \mathbf{F}_2$. The Walsh coefficient $W_f(u)$ (also known as a spectral coefficient), $u \in \mathbf{F}_2^n$, is defined as the real-valued sum:

$$W_f(u) = \sum_{x \in \mathbf{F}_2^n} (-1)^{f(x)+<x,u>}.$$

The plateaued Boolean function is the Boolean function whose Walsh coefficients take values $\{0, \pm 2^c\}$ for some integer c. The given set of Walsh coefficients defines the Boolean function uniquely. If the spectrum support of a plateaued Boolean function f is known (i. e. the set of all vectors $u \in \mathbf{F}_2^n$ such that $W_f(u) \neq 0$) then only signs of all Walsh coefficients are known, so the plateaued Boolean function is not defined uniquely. For the majority of spectrum supports S including the full space $\mathbf{F}_2^n$, n even, $n > 8$, the number of plateaued functions with this spectrum support S is unknown whereas for some specific families the number of functions with such spectrum support was found (see for example [1]). We present some such constructions of spectra and analyse their symmetries. Also we discuss the problem of possible values of a rank (or an affine rank) for given spectrum supports of plateaued Boolean functions. This problem earlier was studied in [2, 3] and recently the new upper bound for an arbitrary Boolean function with the given cardinality of a spectrum support (also known as a sparsity) was obtained in [4].

References

[1] A. O. Logachev, On a recursive class of plateaued Boolean functions. *Discrete Math. Appl.* **20(5-6)** (2010) 537–551.

[2] Yu. V. Tarannikov, On values of the affine rank of the support of spectrum of a plateaued function. *Diskr. Mat.* **18(3)** (2006) 120–137.

[3] C. Carlet, P. Charpin, Cubic Boolean functions with highest resiliency. *Information Theory, IEEE Transactions* (2005) **51(2)** 562–571.

[4] S. Sanyal, Near-Optimal Upper Bound on Fourier Dimension of Boolean Functions in Terms of Fourier Sparsity. InAutomata, Languages, and Programming, Springer, Berlin, Heidelberg, 2015 Jul 6, 1035–1045.