

## On the local structure of Mathon distance-regular graphs

L. Yu. Tsioukina

*N.N. Krasovskii Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia*

l.tsioukina@gmail.com

We study the structure of local graphs of Mathon distance-regular graphs of even valency. This is motivated by the general problem of classification of distance-regular graphs whose local graphs are strongly regular with the second eigenvalue at most a fixed natural number  $n$ . We describe several infinite series of locally  $\Delta$ -graphs of this family, where  $\Delta$  is a pseudogeometric graph for  $pG_l(s, l)$ , a strongly regular graph that is a union of some affine polar graphs of type “–”, or a rank-3 graph, realizable by the van Lint-Shrijver construction.

**Theorem.** *Let  $q = 2^{2t} > 2$  and assume that  $r > 1$  divides  $q - 1$ . Let  $M(q, r)$  denote a Mathon distance-regular graph with intersection array  $\{q, (r - 1)(q - 1)/r, 1; 1, (q - 1)/r, q\}$  and let  $\Delta$  be a local graph of  $M(q, r)$ . Then  $\Delta$  is arc-transitive and the following assertions hold.*

- (1) *If  $r$  divides  $2^t + 1$ , then either*
  - (i)  *$r = 2^t + 1$  and  $\Delta$  is a union of  $2^t$  isolated  $2^t$ -cliques, or*
  - (ii)  *$r < 2^t + 1$  and  $\Delta$  is a strongly regular graph with parameters  $(2^{2t}, (2^t + 1)(2^t - 1)/r, ((2^t + 1)/r - 1)((2^t + 1)/r - 2) + 2^t - 2, (2^t + 1)((2^t + 1)/r - 1)/r)$ .*
- (2) *If  $t$  is even and  $r$  divides  $2^{t/2} + 1$ , then either*
  - (i)  *$r = 2^{t/2} + 1$  and  $\Delta$  is a strongly regular graph with parameters  $(2^{2t}, (2^{t/2} - 1)(2^t + 1), 2^{t/2} - 2, 2^{t/2}(2^{t/2} - 1))$  that is isomorphic to  $\text{VO}^-(4, 2^{t/2})$ , or*
  - (ii)  *$r < 2^{t/2} + 1$  and  $\Delta$  is a strongly regular graph with parameters  $(2^{2t}, z(2^{t/2} - 1)(2^t + 1), z(2^{t/2} - 1)(3 + z(2^{t/2} - 1)) - 2^t, z(2^{t/2} - 1)(1 + z(2^{t/2} - 1)))$  which is a union of  $z = (2^{t/2} + 1)/r$  graphs that are isomorphic to  $\text{VO}^-(4, 2^{t/2})$ .*
- (3) *If  $r$  is a prime divisor of  $q - 1$ ,  $2$  is a primitive root modulo  $r$  and  $(r - 1)$  divides  $2t$ , then  $\Delta$  is a rank-3 graph with parameters  $(2^{2t}, (2^{2t} - 1)/r, (2^{2t} - 3r + 1 + \epsilon(r - 1)(r - 2)2^t)/r^2, (2^{2t} - r + 1 - \epsilon(r - 2)2^t)/r^2)$  that is realized by the van Lint-Shrijver construction, where  $\epsilon = (-1)^{2t/(r-1)+1}$ .*

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### References

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