Critical phenomena in random graphs

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In first model we consider random nondirected networks subject to dynamics conserving vertex degrees and study, analytically and numerically, equilibrium three-vertex motif distributions in the presence of an external field h coupled to one of the motifs. For small h, the numerics is well described by the chemical kinetics for the concentrations of motifs based on the law of mass action. For larger h, a transition into some trapped motif state occurs in Erdős-Renyi networks.

The second model is devoted to an equilibrium ensemble of large Erdős-Renyi topological random networks with two types of vertices, black and white, and fixed vertex degree prepared randomly with the bond connection probability, p. The system energy is a sum of all unicolor triples (either all black or all white), weighted with a chemical potential of triples, μ. Minimizing the system energy, we see at any positive μ the formation of two predominantly unicolor clusters, linked by a "string" of N_{bw} black-white bonds. The system exhibits a critical behavior manifested in emergence of a wide plateau on the $N_{bw}(\mu)$-curve. We have proposed an explanation of plateau formation in terms of statistical physics, relevant to spinodal decomposition of 1st order phase transitions.