On products of submodular subgroups of finite groups

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Throughout this report, all groups are finite. Recall that a subgroup M of a group G is said to be modular in G if M is a modular element of the lattice of all subgroups of G [1]. It means that the following conditions are fulfilled:

(1) $\langle X, M \cap Z\rangle = \langle X, M\rangle \cap Z$ for all $X \leq G, Z \leq G$ such that $X \leq Z$, and

(2) $\langle M, Y \cap Z\rangle = \langle M, Y\rangle \cap Z$ for all $Y \leq G, Z \leq G$ such that $M \leq Z$.

In the paper [2] I. Zimmermann introduced the notion of a submodular subgroup which generalizes the notion of a subnormal subgroup. Recall that a subgroup H of a group G is said to be submodular in G [2], if there exists a chain of subgroups $H = H_0 \leq H_1 \leq \ldots \leq H_{s-1} \leq H_s = G$ such that H_{i-1} is a modular subgroup in H_i for $i = 1, \ldots, s$.

In [3] the class $sm\mathfrak{U}$ of all groups with submodular Sylow subgroups was investigated and some of its properties were found. For instance, it was proved in [3] that $sm\mathfrak{U}$ forms a hereditary saturated formation, its local function was found, criteria of the membership of a group to the class $sm\mathfrak{U}$ were established.

This report is devoted to the further development of results of the paper [3]. In particular, we obtained the following result.

Theorem. *Let G be a group, $G = G_1G_2$ be a product of submodular subgroups G_1 and G_2 such that $G_i \in sm\mathfrak{U}$, $i = 1, 2$, and $(|G : G_1|, |G : G_2|) = 1$. Then $G \in sm\mathfrak{U}$.*

References

[1] R. Schmidt, *Subgroup Lattices of Groups*, Walter de Gruyter, Berlin etc, 1994.

[2] I. Zimmermann, Submodular Subgroups in Finite Groups. *Math. Z.* **202** (1989) 545–557.

[3] V. A. Vasilyev, Finite groups with submodular Sylow subgroups. *Siberian Mathematical Journal* **56(6)** (2015) 1019–1027.