On Fourier decomposition of Preparata-like codes in the graph of the hypercube

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Given a graph G, a subset $C \subseteq V(G)$ is called a *perfect code (with distance* 3*)* if each vertex of G is at distance no more than 1 from exactly one code vertex. We consider the graph Q^n of n-dimensional binary hypercube. In this case perfect codes exist for every n of form $n = 2^t - 1$ and do not exist for any other n. In the case $n = 4^t - 1$ there exist *Preparata-like codes* (we call them as *Preparata codes*), which are defined as the codes of distance 5 and size $2^{n+1}/(n+1)^2$. Every Preparata code P is included in a unique perfect code $C(P)$ [1]; we denote this perfect code as $C(P)$.

Let f_D^i denotes the orthogonal projection of the characteristic function of the set D to the i-th eigensubspace of the graph of Q^n. (For any function its Fourier transform is defined as the collection of its orthogonal projections to the eigenfunctions $\varphi^\alpha(x) = (-1)^{\langle \alpha, x\rangle},\ \alpha, x \in Q^n$.)

It is known that for any perfect code C its characteristic function χ_C can be represented as follows:

$$\chi_C = 1/(n+1) + f_C^{(n+1)/2}.$$

Analogously, for an arbitrary Preparata code P we have:

$$\chi_P = 1/(n+1) + f_P^{(n+1)/2} + f_P^k + f_P^h,$$

where $k = (n+1)/2 - \sqrt{n+1}/2$ and $h = (n+1)/2 + \sqrt{n+1}/2$.

Theorem. *Let P be an arbitrary Preparata code in the graph of n-dimensional binary hypercube. Then*

$$f_P^{(n+1)/2} = \frac{2}{n+1} f_{C(P)}^{(n+1)/2}.$$

It is known that an i-component R of an arbitrary Preparata code can be extended to the i-component $S(R)$ of the perfect code $C(P)$. Then we have as a corollary that $f_R^{(n+1)/2} = \frac{2}{n+1} f_{S(R)}^{(n+1)/2}$.

Acknowledgments. The work was funded by the Russian science foundation (grant 14-11-00555)

References

[1] G. V. Zaitsev, V. A. Zinoviev, and N. V. Semakov, Interrelation of Preparata and Hamming codes and extension of Hamming codes to new double-error-correcting codes. In P. N. Petrov and F. Csaki, editors *Proc. 2nd Int. Symp. Information Theory, Tsahkadsor, Armenia, USSR, 1971*, 257–264, Budapest, Hungary, 1973. Akademiai Kiado.