On Fourier decomposition of Preparata-like codes in the graph of the hypercube

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Given a graph $G$, a subset $C \subseteq V(G)$ is called a perfect code (with distance 3) if each vertex of $G$ is at distance no more than 1 from exactly one code vertex. We consider the graph $Q^n$ of $n$-dimensional binary hypercube. In this case perfect codes exist for every $n$ of form $n = 2^t - 1$ and do not exist for any other $n$. In the case $n = 4^t - 1$ there exist Preparata-like codes (we call them as Preparata codes), which are defined as the codes of distance 5 and size $2^{n+1}/(n+1)^2$. Every Preparata code $P$ is included in a unique perfect code $C(P)$ [1]; we denote this perfect code as $C(P)$.

Let $f_D$ denotes the orthogonal projection of the characteristic function of the set $D$ to the $i$-th eigensubspace of the graph of $Q^n$. (For any function its Fourier transform is defined as the collection of its orthogonal projections to the eigenfunctions $\phi_{\alpha}(x) = (-1)^{\langle \alpha, x \rangle}$, $\alpha, x \in Q^n$.)

It is known that for any perfect code $C$ its characteristic function $\chi_C$ can be represented as follows:

$$\chi_C = 1/(n + 1) + f_C^{(n+1)/2}.$$  

Analogously, for an arbitrary Preparata code $P$ we have:

$$\chi_P = 1/(n + 1) + f_P^{(n+1)/2} + f_P^k + f_P^h,$$

where $k = (n + 1)/2 - \sqrt{n + 1}/2$ and $h = (n + 1)/2 + \sqrt{n + 1}/2$.

**Theorem.** Let $P$ be an arbitrary Preparata code in the graph of $n$-dimensional binary hypercube. Then

$$f_P^{(n+1)/2} = \frac{2}{n + 1} f_C^{(n+1)/2}.$$  

It is known that an $i$-component $R$ of an arbitrary Preparata code can be extended to the $i$-component $S(R)$ of the perfect code $C(P)$. Then we have as a corollary that $f_R^{(n+1)/2} = \frac{2}{n + 1} f_{S(R)}^{(n+1)/2}$.

**Acknowledgments.** The work was funded by the Russian science foundation (grant 14-11-00555)

**References**