The lit-only σ-game and some mathematics around

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Let G be a graph. We regard the vertex space, which is the power set of the vertex set of G, as a vector space over the binary field $\mathbb{F}_2$. For each vertex v in G, let T_v be the endomorphism of the vertex space mapping the vertex v to $v + \mathrm{N}_G(v)$, where $\mathrm{N}_G(v)$ is the neighbourhood of v in G, and mapping the vertex w to w itself for all other vertices w in G. In other words, T_v can be written as $\mathrm{id} + \mathrm{N}_G(v)v^*$, where v^* is the Kronecker function for v. Note that T_v is a transvection if v is not a loop vertex, namely if $v \notin \mathrm{N}_G(v)$, while T_v is a projection if v is a loop vertex, namely if $v \in \mathrm{N}_G(v)$.

In the case that G is loopless, the set of all T_v's, where v runs over the vertex set of G, generates a group, called the lit-only group of the graph G. We prove that the lit-only group is a semidirect product of a classical group over $\mathbb{F}_2$ and an elementary abelian 2-group, and we give explicit description of the orbits of the corresponding group action.

In the case that G contains loops, the set of all T_v's, which consists of possible transvections and some projections, generates a monoid. We describe the orbits of this monoid action.