Let $L = SL_3(2) \cong PSL_3(2)$ and let $V_3$ be the natural module of $L$, so that $V_3$ is a 3-dimensional $GF(2)$-space of which $L$ is the automorphism group, and let $V_1$ be the trivial 1-dimensional $GF(2)$-module of $L$. Let $0 \neq v \in V_3$. Denote by $H$ the stabilizer of $v$ in $L$.

(1) Calculate the number of subgroups isomorphic to $L$ in the semidirect product of $V_3$ and $L$ with respect to the natural action of $L$ on $V_3$.

(2) Show that there is the unique indecomposable extension of $V_3$ by $V_1$ (that is a 4-dimensional $GF(2)$-module whose only proper submodules are isomorphic to $V_3$ and the unique module $V_1 \setminus V_3$).

(3) Is there an uniserial module $V_1 \setminus V_3 \setminus V_1$?

(4) What is the submodule structure of

(a) the $GF(2)$-permutation module of $L$ on the cosets of subgroups $H$, $H'$, $F = F(7, 3)$ (which is a Frobenius group with the core of order 7 and a complement of order 3), $F'$ (which is a cyclic group of order 7)?

(b) $V_3 \otimes V_3$, $V_3 \otimes V_3^*$ (where $V_3^*$ is the dual of $V_3$)?

(5) Let $W = V_3^* \setminus V_3$ be the direct summand of the permutational module of $L$ on the cosets of $H$. Let $X$ be the semidirect product $W \ltimes L$. Calculate the number of subgroups of $X$ which are isomorphic to $L$. 