On Eulerian Identities in $UT_n(\mathbb{Q})$

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One of the known methods of building identities in matrix rings deals with eulerian multigraphs. A multigraph is a graph which is permitted to have multiple oriented edges with same end nodes, and loops - edges with same start and end node. Suppose, $\mathcal{G}$ is an eulerian graph with n edges and $\Pi(\mathcal{G})$ is the set of all its eulerian paths. Every such path π we consider to be a permutation of set $\{1, 2, \ldots, n\}$, by $sgn(\pi)$ we denote the sign of this transposition. An *eulerian polynomial of the graph* $\mathcal{G}$ is the polynomial:

$$P_{\mathcal{G}} = \sum_{\pi \in \Pi(\mathcal{G})} sgn(\pi) x_{\pi(1)} \cdots x_{\pi(n)}.$$

The examples of eulerian polynomials are well known standard polynomials

$$\mathcal{S}_n = \sum_{\pi \in S_n} sgn(\pi) x_{\pi(1)} \cdots x_{\pi(n)},$$

which are built from graphs with one vertex and n loops. The identity $f = 0$ is called eulerian, if f is an eulerian polynomial of some graph.

The identity basis of 2×2 matrices ring from eulerian polynomials is already found by M. Domokos in [1]. The following result deals with ring of upper triangular matrices $UT_n(\mathbb{Q})$, the identity basis of such ring from non-eulerian polynomials is also found by Yu. N. Maltsev in [2].

Theorem. *In the class of rings with 1, the eulerian polynomial*

$$P_{UT_n}(x_1, \ldots, x_n, y_1, \ldots, y_n, e_1, \ldots, e_{n-1}) = [x_1, y_1]e_1[x_2, y_2]e_2 \ldots e_{n-1}[x_n, y_n]$$

forms basis of the ring $UT_n(\mathbb{Q})$. The polynomial P_{UT_n} is built from graph G_{UT_n} displayed on fig.1.

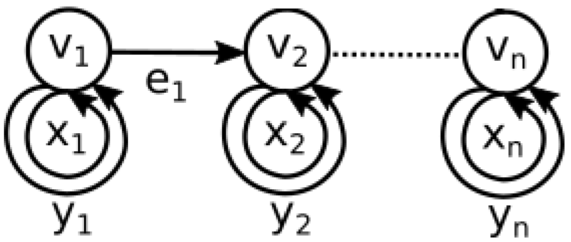

Figure 1: G_{UT_n}

References

[1] M. Domokos, Eulerian polynomial identities and algebras satisfying a standard identity. *J.Algebra* **169** (1994) 913–928.

[2] Yu. N. Maltsev, A basis for the identities of the algebra of upper triangular matrices (Russian). *Algebra i Logika* **10** (1971) 393–400. Translation: *Algebra and Logic* **10** (1971) 242–247.