A family of regular coherent non-Schurian graphs, related to extremal graph theory

Matan Ziv-Av
Ben-Gurion University of the Negev, Beer Sheva, Israel
matan@svgalib.org

This is joint work with Mikhail Klin and Guillermo Pineda-Villavicencio

In this lecture we attempt to establish some natural links between Algebraic and Extremal graph theories (AGT and EGT, respectively).

We start by considering significant concepts in AGT such as coherent configurations and their particular case, association schemes. There exists an efficient, polynomial time, algorithm which for a given graph Γ, calculates the smallest coherent configuration $W(\Gamma)$ containing Γ as a union of basic graphs. Nowadays, $W(\Gamma)$ is called the WL-closure of Γ (in honor of Weisfeiler and Leman). Recently, this subject became more popular due to its links with the graph isomorphism problem.

We call a graph Γ coherent if it is a basic graph of $W(\Gamma)$. The coherent configuration $W(\Gamma)$ is called Schurian if it coincides with the centralizer algebra of $Aut(\Gamma)$, otherwise, $W(\Gamma)$ is non-Schurian.

Many extremal graphs have a rich automorphism group, in particular they are coherent and Schurian. Moore graphs of valencies 3 and 7, as well as the cages which are incidence graphs of classical generalized polygons are examples of such nice objects.

We will try to explain why in the framework of AGT the above mentioned classes of extremal graphs should be naturally substituted by coherent and non-Schurian graphs appearing as a subject of EGT.

A few sporadic examples will be considered together with a family of regular bipartite graphs on $2(q^2-1)$ vertices, $q \geq 3$ is a prime power. These graphs have valency q, diameter 4, and a rank 6 WL-closure with valencies 1, q, $q(q-1)$, $q(q-2)$, $q-1$, $q-2$.