

On the spectra of automorphic extensions of finite simple exceptional groups of Lie type*Maria Zvezdina**Sobolev Institute of Mathematics SB RAS, Novosibirsk, Russia*

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Given a finite group G , the *spectrum* of G is the set of orders of its elements. This set is denoted by $\omega(G)$. Groups are called *isospectral* if their spectra are equal. The number of pairwise non-isomorphic groups isospectral to a group G is denoted by $h(G)$. A group G is called *recognizable by spectrum* if $h(G) = 1$, and *almost recognizable* if $h(G)$ is finite. We say that the *recognition problem* is solved for a group G , if $h(G)$ is known, and if $h(G)$ is finite, the description of all pairwise non-isomorphic groups isospectral to G is given.

Let S be a finite simple exceptional group of Lie type and $S \neq {}^3D_4(2)$. Then S is known to be almost recognizable. Moreover, every finite group isospectral to S is isomorphic to some group G such that $S \leq G \leq \text{Aut}(S)$ [1, Theorem 1]. So, in order to solve the recognition problem for a group S , it suffices to describe all groups G such that $\omega(G) = \omega(S)$ and $S \leq G \leq \text{Aut}(S)$. Such descriptions already exist for all exceptional groups except for the groups of types E_6 and E_7 . Let us denote groups of type E_6 by $E_6^\varepsilon(q)$, $\varepsilon \in \{+, -\}$, where $E_6(q) = E_6^+(q)$ and ${}^2E_6(q) = E_6^-(q)$. We complete the investigation of recognition problem for finite simple exceptional groups of Lie type by following results:

Theorem 1. *Let S be a finite simple exceptional group $E_6^\varepsilon(q)$, where q is a power of prime p , and let $S < G \leq \text{Aut}(S)$. Then $\omega(G) = \omega(S)$ if and only if G is an extension of S by a field automorphism, G/S is a 3-group, 3 divides $q - \varepsilon 1$ and $p \notin \{2, 11\}$.*

Theorem 2. *Let S be a finite simple exceptional group $E_7(q)$, where q is a power of prime p , and let $S < G \leq \text{Aut}(S)$. Then $\omega(G) = \omega(S)$ if and only if G is an extension of S by a field automorphism, G/S is a 2-group and $p \notin \{2, 13, 17\}$.*

These results also complete the study of recognition problem for finite simple groups of Lie type over fields of characteristic 2 (results for classical groups are given in [2]).

References

- [1] A. V. Vasil'ev, A. M. Staroletov, Almost recognizability by spectrum of simple exceptional groups of Lie type. *Algebra and Logic* **53(6)** (2014) 433–449.
- [2] A. V. Vasil'ev, M. A. Grechkoseeva, Recognition by spectrum for simple classical groups in characteristic 2. *Sib. Math. J.* **56(6)** (2015) 1009–1018.