On group density function

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Let $G = \langle g_1, ..., g_n \rangle$ be a group and $\mathfrak{N} = \{g_1, ..., g_n\}$ be a set of its generators. The number l for $g \in G$ is called *length* of g if shortest presentation of g such as product of generators contains l elements. The function $F_{(G,\mathfrak{N})}(l)$ is called a group *growth function* if $F_{(G,\mathfrak{N})}(l)$ is equal to number of elements with length at most l.

Let

$$P_{(G,\mathfrak{N})}(l) = \begin{cases} 1, & \text{if } l = 0, \\ F_{(G,\mathfrak{N})}(l) - F_{(G,\mathfrak{N})}(l-1) & \text{if } l > 0. \end{cases}$$

be a group density function of G on a set of generators $\mathfrak{N}$.
Question. Let G and H be groups with set of generators $\mathfrak{N}$ and $\mathfrak{M}$ respectively such that $P_{(G,\mathfrak{N})}(l) = P_{(H,\mathfrak{M})}(l)$. Is $G \simeq H$?

Proposition 1.

Let G and H be groups, $\mathfrak{N} = \{g_1, ..., g_n\}$ and $\mathfrak{M} = \{b_1, ..., b_m\}$ be sets of generators of G and H respectively. If $P_{(G,\mathfrak{N})}(l) = P_{(H,\mathfrak{M})}(l)$ then $|\mathfrak{N}| = |\mathfrak{M}|$.

Proposition 2.

Let G and H be groups, $\mathfrak{N} = \{g_1, ..., g_n\}$ and $\mathfrak{M} = \{b_1, ..., b_m\}$ be sets of generators of G and H respectively. If $P_{(G,\mathfrak{N})}(l) = P_{(H,\mathfrak{M})}(l)$ then $|G| = |H|$.

Definition 3

Let $G = \langle g_1, ..., g_n \rangle$ be group with a set of generators $\mathfrak{M} = \{g_1, ..., g_n\}$. The set $\mathfrak{M}$ is called **minimal**, if $\langle \mathfrak{M} \setminus g_i \rangle \neq G$ for all $i \in \{1, ..., n\}$.

Definition 4.

Let $G = \langle g_1, ..., g_n \rangle$ be a group with a set of generators $\mathfrak{M} = \{g_1, ..., g_n\}$. The set $\mathfrak{M}$ is called **independent**, if $\langle \mathfrak{M} \setminus \{g_i\} \rangle \cap \langle g_i \rangle$ is trivial for all $i \in \{1, ..., n\}$.

Theorem 1.

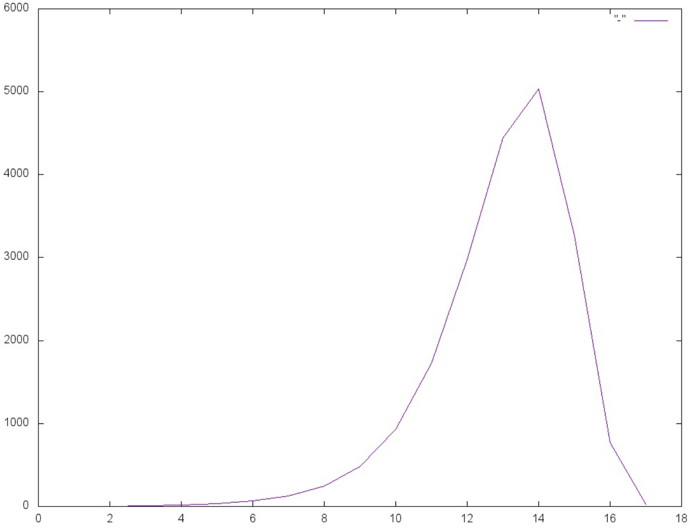
Let $G = \langle \mathfrak{N} \rangle$ and $H = \langle \mathfrak{M} \rangle$ be abelian p-groups, $\mathfrak{N}$ and $\mathfrak{M}$ be their sets of generators respectively. If $P_{(G,\mathfrak{N})}(l) = P_{(H,\mathfrak{M})}(l)$ then $G \simeq H$.

Conjecture. Let G and H be finite simple non abelian groups, $\mathfrak{N}$ and $\mathfrak{M}$ be independent sets of their generators respectively. If $P_{(G,\mathfrak{N})}(l) = P_{(H,\mathfrak{M})}(l)$ then $G \simeq H$.

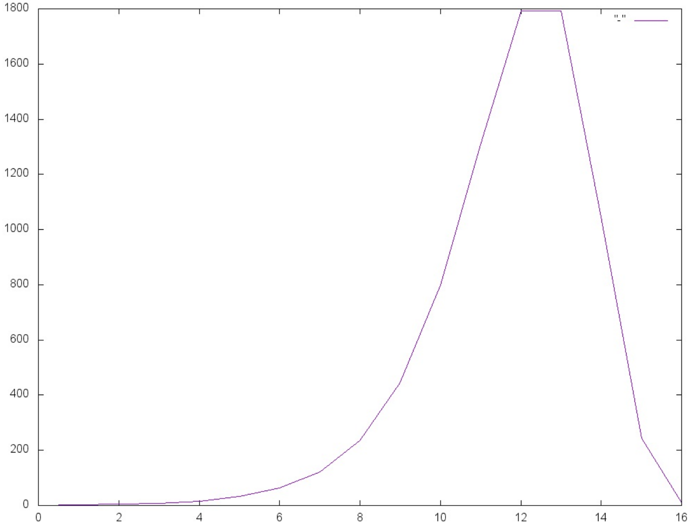
Theorem 2.

If $A_8 = \langle \mathfrak{N} \rangle$, $L_3(4) = \langle \mathfrak{M} \rangle$, and $|\mathfrak{N}| = |\mathfrak{M}| = 2$. Then $P_{(A_8,\mathfrak{N})}(l) \neq P_{(L_3(4),\mathfrak{M})}(l)$.

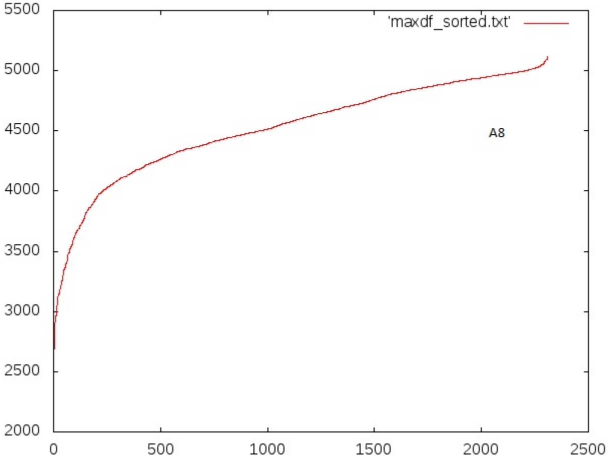
Density function of A8



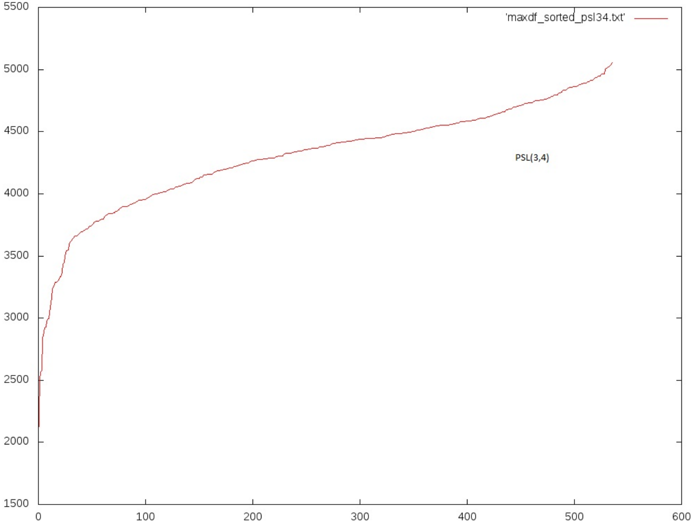
Density function of M(1,1)



Density functions maximal values of A8 sorted by increasing



Density functions maximal values of PSL(3,4) sorted by increasing



Statistics

	A8	PSL(3,4)	M(1,1)
Group Order	20160	20160	7920
Generation Pairs	150151680	175633920	25652880
Density functions	150151680	175633920	25652880
Different density functions	2311	536	1693
Possible frequencies	20160 40320 80640	120960 241920 483840	7920 15840