COMPLEXITY OF HYPERBOLIC 3-MANIFOLDS

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The most useful approach to a classification of 3-manifolds is the complexity theory founded by S. Matveev [1]. Unfortunately, exact values of complexity are known for few infinite series of 3-manifold only.

We present the results on complexity for two infinite series of hyperbolic 3-manifolds with boundary. The first is a family of Paoluzzi – Zimmermann manifolds from [1], and the second is its analogy defined in [4]. These manifolds are constructing as follows.

For every $n \geq 3$ consider an $n$-gonal bipyramid $B_n$, the union of pyramids $NL_0L_1 \ldots L_{n-1}$ and $SL_0L_1 \ldots L_{n-1}$ along the common $n$-gonal base $L_0L_1 \ldots L_{n-1}$. Let $k$ be such integer that $0 \leq k < n$. The first family corresponds to the case $\gcd(n, 2 - k) = 1$, and the second – to the case $\gcd(n, 2 - k) = 2$. Let us identify the faces of $B_n$ in pairs: for each $i = 0, \ldots, n - 1$ the face $L_iL_{i+1}N$ gets identified with the face $SL_iL_{i+k+1}$ by a homeomorphism of faces. (indices are taken mod $n$ and the vertices are glued together in the order in which they are written).

Denote the resulting identification spaces by $M_{n,k}^*$. It is an orientable pseudomanifold with one singular point. Cutting of a cone neighborhood of the singular point from $M_{n,k}^*$ we get a compact manifold $M_{n,k}$ with one boundary component.

Denote by $c(M_{n,k})$ the Matveev’s complexity of $M_{n,k}$ which is defined as the minimum possible number of true vertices of an almost simple spine of $M_{n,k}$.

**Theorem.** [3, 4] Suppose that $\gcd(n, 2 - k) = 1$ or $\gcd(n, 2 - k) = 2$. Then for every integer $n \geq 6$ we have $c(M_{n,k}) = n$.

**References**


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