Intersection of conjugate solvable subgroups in finite groups

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Assume that a finite group G acts on a set Ω. An element $x \in \Omega$ is called a *regular point* if $|xG| = |G|$, i.e. if the stabilizer of x is trivial. Define the action of the group G on Ω^k by

$$g : (i_1, \dots, i_k) \mapsto (i_1 g, \dots, i_k g).$$

If G acts faithfully and transitively on Ω, then the minimal number k such that the set Ω^k contains a G-regular point is called the *base size* of G and is denoted by $b(G)$. For a positive integer m the number of G-regular orbits on Ω^m is denoted by $Reg(G, m)$ (this number equals 0 if $m < b(G)$). If H is a subgroup of G and G acts by the right multiplication on the set Ω of right cosets of H then G/H_G acts faithfully and transitively on the set Ω. (Here $H_G = \cap_{g \in G} H^g$.) In this case we denote $b(G/H_G)$ and $Reg(G/H_G, m)$ by $b_H(G)$ and $Reg_H(G, m)$ respectively.

Thus $b_H(G)$ is the minimal number k such that there exist $x_1, \dots, x_k \in G$ with $H^{x_1} \cap \dots \cap H^{x_k} = H_G$.

Consider Problem 17.41 b) from "Kourovka notebook"[1]:

Let H be a solvable subgroup of finite group G that has no nontrivial solvable normal subgroups. Do there always exist five conjugates of H whose intersection is trivial?

The problem is reduced to the case when G is almost simple in [2]. Specifically, it is proved that if for each almost simple group G and solvable subgroup H of G inequality $Reg_H(G, 5) \geq 5$ holds then for each finite nonsolvable group G and maximal solvable subgroup H of G inequality $Reg_H(G, 5) \geq 5$ holds.

In the talk we discuss the recent progress in the solution of the problem.

References

[1] *The Kourovka Notebook: Unsolved problems in group theory*, 18 ed., arXiv:1401.0300.

[2] E. P. Vdovin, *On the base size of a transitive group with solvable point stabilizer*, Journal of Algebra and Application **11** (2012), no. 1, 1250015 (14 pages)

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