On element orders of finite almost simple groups

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The orders of elements are among the most basic concepts relating to a finite group, so it seems quite natural to ask what element orders the finite almost simple groups have, or more precisely, given a finite nonabelian simple group S and $g \in \operatorname{Aut} S$, what the orders of elements in the coset gS are.

Another motivation for this question is the following recent result: if S is a simple alternating group other than A_6 and A_{10}, or a sporadic group other than J_2, or a simple exceptional group of Lie type other than ${}^3D_4(2)$, or a simple classical group of dimension at least 62, then every finite group whose set of element orders is equal to that of S is an almost simple group with socle S (see [1] for details). Moreover, if S is alternating or sporadic (and not A_6, A_{10}, J_2), then S is uniquely determined by the set of its element orders in the class of finite groups. But if S is a group of Lie type, then in general there can be a nontrivial almost simple extension of S with the same set of element orders, so further work is required to determine such extensions.

In this talk, we discuss ways to calculate the orders of elements of a coset gS and give a complete answer to the question what almost simple groups have the same set of element orders as its socle.

References

[1] A.V. Vasil'ev and M.A. Grechkoseeva, *On the structure of finite groups isospectral to finite simple groups*, J. Group Theory **18** (2015), no. 5, 741–759.

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