

Butson-Hadamard matrices in association schemes of class 6 on Galois rings of characteristic 4

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A complex Hadamard matrix is a square matrix W of order n which satisfies $W\overline{W}^\top = nI$ and all of whose entries are complex numbers of absolute value 1. A complex Hadamard matrix is said to be Butson-type, if all of its entries are roots of unity. In an earlier work [2], we proposed a method to classify symmetric complex Hadamard matrices belonging to the Bose–Mesner algebra of a symmetric association scheme.

Galois rings have been used to construct certain association schemes (see [3, 4, 5]), and certain properties of association schemes obtained from Galois rings have been investigated in [1].

In this talk, we give a construction of a nonsymmetric association scheme \mathfrak{X} of class 6 on the Galois ring of characteristic 4, and classify hermitian complex Hadamard matrices belonging to the Bose–Mesner algebra of \mathfrak{X} . We show that such a matrix is necessarily a Butson-type matrix whose entries are 4-th roots of unity. One of the family of such matrices actually belongs to the smaller Bose–Mesner algebra of a class 3 fusion scheme. These fusion schemes are the only family (parametrically) of class 3 nonsymmetric schemes whose Bose–Mesner algebra contains a (non-real) hermitian complex Hadamard matrix.

This is based on joint work with Takuya Ikuta.

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