Федеральное государственное бюджетное учреждение науки
Институт математики им. С. Л. Соболева
Сибирского отделения Российской академии наук

Федеральное государственное автономное образовательное
учреждение высшего образования
«Новосибирский национальный исследовательский
государственный университет»

Международный семинар

ТЕОРИЯ ГРУПП И АЛГЕБРАИЧЕСКАЯ КОМБИНАТОРИКА

16–21 июля 2017

Тезисы докладов

РФФИ

Конференция проведена при финансовой поддержке
Российского фонда фундаментальных исследований
(код проекта 17-01-20308)

N* Новосибирский государственный университет
*НАСТОЯЩАЯ НАУКА

Международный математический центр
Новосибирского государственного университета

Новосибирск, 2017

Sobolev Institute of Mathematics

Novosibirsk State University

Workshop on

GROUP THEORY AND ALGEBRAIC COMBINATORICS

16–21 July 2017

Collection of Abstracts

Supported by
Russian Foundation for Basic Research
(grant 17-01-20308)

International Mathematical Center
of Novosibirsk State University

Novosibirsk, 2017

Contents

Invited talks

Generating simple groups and their subgroups

Tim Burness

It is well known that every finite simple group can be generated by two elements and this leads to a wide range of problems that have been the focus of intensive research in recent years, such as random generation, $(2,3)$-generation and so on. In this talk I will report on recent joint work with Martin Liebeck and Aner Shalev on similar problems for subgroups of (almost) simple groups, focussing on maximal and second maximal subgroups. In particular, we prove that every maximal subgroup of a simple group can be generated by four elements (this is best possible) and we show that the problem of determining a bound on the number of generators for second maximal subgroups depends on a formidable open problem in Number Theory. I will highlight some applications to primitive permutation groups and subgroup growth. Finally, I will report on some related work in progress concerning a new notion of depth for finite groups, focussing on results for simple groups.

References

[1] T.C. Burness, M.W. Liebeck and A. Shalev, *Generation and random generation: from simple groups to maximal subgroups*, Advances in Mathematics **248** (2013), 59–95.

[2] T.C. Burness, M.W. Liebeck and A. Shalev, *Generation of second maximal subgroups and the existence of special primes*, submitted (arxiv:1611.06196).

[3] T.C. Burness, M.W. Liebeck and A. Shalev, *The depth of a finite simple group*, in preparation.

School of Mathematics, University of Bristol, United Kingdom

E-mail address: t.burness@bristol.ac.uk

Majorana Representations of the Symmetric Groups

Clara Franchi

Majorana representations have been introduced by Alexander A. Ivanov to give an axiomatic framework for the study of the representations of the Monster group, and its $2A$-generated subgroups, on the Conway-Norton-Griess algebra. I shall present some recent results on Majorana Representations of the symmetric groups obtained in a joint work with A.A. Ivanov and M. Mainardis.

Università Cattolica del Sacro Cuore, Italy

E-mail address: clara.franchi@unicatt.it

On element orders of finite almost simple groups

Maria Grechkoseeva

The orders of elements are among the most basic concepts relating to a finite group, so it seems quite natural to ask what element orders the finite almost simple groups have, or more precisely, given a finite nonabelian simple group S and $g \in \operatorname{Aut} S$, what the orders of elements in the coset gS are.

Another motivation for this question is the following recent result: if S is a simple alternating group other than A_6 and A_{10}, or a sporadic group other than J_2, or a simple exceptional group of Lie type other than ${}^3D_4(2)$, or a simple classical group of dimension at least 62, then every finite group whose set of element orders is equal to that of S is an almost simple group with socle S (see [1] for details). Moreover, if S is alternating or sporadic (and not A_6, A_{10}, J_2), then S is uniquely determined by the set of its element orders in the class of finite groups. But if S is a group of Lie type, then in general there can be a nontrivial almost simple extension of S with the same set of element orders, so further work is required to determine such extensions.

In this talk, we discuss ways to calculate the orders of elements of a coset gS and give a complete answer to the question what almost simple groups have the same set of element orders as its socle.

References

[1] A.V. Vasil'ev and M.A. Grechkoseeva, *On the structure of finite groups isospectral to finite simple groups*, J. Group Theory **18** (2015), no. 5, 741–759.

Sobolev Institute of Mathematics
E-mail address: gma@math.nsc.ru

Locally defined algebras and their automorphisms

Jonathan Hall

Commutative, associative, and Lie algebras are defined locally in the sense that the identities that characterize them can be checked in subalgebras generated by a small number of elements. Theorems like that of Sakuma characterizing subalgebras of Griess algebras generated by two Ising vectors suggest that we consider algebras presented in terms of certain finitely generated subalgebras. We can then consider the circumstances under which these algebras or their quotients have global properties, such as large automorphism groups and invariant forms.

Michigan State University, USA

Modular standard modules of association schemes

Akihide Hanaki

We are trying to establish theory of modular representations of association schemes. Modular representations mean representations of adjacency algebras of association schemes over a positive characteristic field. In general, if two association schemes are algebraically isomorphic, namely they have the same intersection numbers, then the adjacency algebras over an arbitrary coefficient ring are isomorphic and representations are same. However, an association scheme have the special representation, the standard representation. Let (X, S) be an association scheme, and let R be a commutative ring with 1. We denote by $M_X(R)$ the full matrix algebra over R, rows and columns of whose matrices are indexed by the set X. The adjacency algebra RS is defined as a subalgebra of $M_X(R)$. Thus the inclusion $RS \to M_X(R)$ is a representation of (X, S), and we call this the *standard representation* over R.

In some papers, for example in [1, 5, 6], p-ranks of some integral matrices were considered and sometimes they could distinguish combinatorial objects with same parameters. In [4], it was pointed out that p-ranks were "shadows" of structures of standard modules.

In this talk, we will consider an extreme case, the case that the standard module is indecomposable. Let F be a field of positive characteristic p, and let G be a finite group. Then it is well known that the following conditions are equivalent :

(1) The group G is a p-group.
(2) The group algebra FG is a local algebra.
(3) The regular FG-module is indecomposable.

For an association scheme (X, S), it is known that

- If (X, S) is a p-scheme, then the adjacency algebra FS is a local algebra [2] and the standard module FX is an indecomposable FS-module [3].

By examples, we know that the conditions are not equivalent for association schemes. However, for schurian schemes, we have the following result.

Theorem 1. *Let F be a field of positive characteristic p, and let (X, S) be a schurian association scheme. Then (X, S) is a p-scheme if and only if the standard module FX is an indecomposable FS-module.*

A schurian association scheme is defined by a transitive permutation group, but the group is not unique, in general. It is easy to see that a transitive permutation p-group defines a schurian p-scheme. As a corollary to Theorem 1, we can show that the converse is also true.

Corollary 2. *Let G be a transitive permutation group. If G defines a schurian p-scheme, then G is a p-group.*

References

[1] A. E. Brouwer and C. A. van Eijl, *On the p-rank of the adjacency matrices of strongly regular graphs*, J. Algebraic Combin. **1** (1992), no. 4, 329–346.

[2] A. Hanaki, *Locality of a modular adjacency algebra of an association scheme of prime power order*, Arch. Math. (Basel) **79** (2002), no. 3, 167–170.

[3] A. Hanaki and O Shimabukuro, *Indecomposable decompositions of modular standard modules for two families of association schemes*, to appear in J. Algebraic Combin.

[4] A. Hanaki and M. Yoshikawa, *On modular standard modules of association schemes*, J. Algebraic Combin. **21** (2005), no. 3, 269–279.

[5] R. Peeters, *Uniqueness of strongly regular graphs having minimal p-rank*, Linear Algebra Appl. **226**/**228** (1995), 9–31.

[6] ———, *On the p-ranks of the adjacency matrices of distance-regular graphs*, J. Algebraic Combin. **15** (2002), no. 2, 127–149.

Faculty of Science, Shinshu University, Japan
E-mail address: hanaki@shinshu-u.ac.jp

Results on meta-thin association schemes

Mitsugu Hirasaka

Let (X, S) be an association scheme where X is a finite set and S is a partition of $X \times X$. We say that (X, S) is *schurian* if S is the set of orbitals of a permutation group of X. Let $H \leq N \trianglelefteq G$ with $N \leq N_G(H)$. Then G acts on G/H by the right multiplication. Here we focus on some class of association schemes, called *meta-thin*, to generalize the set of orbitals of the above action. Since the study of meta-thin association schemes was started in 2002, several sufficient conditions for a meta-thin association scheme to be schurian has been found (see [1],[2] and [3]). In this talk we introduce some basic terminologies to define meta-thin association schemes, we follow up a series of known results according to the chronological order and we show a new result on this topic, which is obtained by a joint work with K. Kim and I. Ponomarenko.

References

[1] M. Hirasaka, P.-H. Zieschang, *Sufficient conditions for a scheme to originate from a group*, J. Combin. Theory Ser. A **104** (2003), no. 1, 17–27.

[2] M. Hirasaka, *On meta-thin association schemes*, Des. Codes Cryptogr. **34** (2005), no. 2–3, 187–201.

[3] P.-H. Zieschang, *On association schemes with thin thin residue*, J. Algebra **322** (2009), no. 1, 54–67.

Pusan National University, Republic of Korea
E-mail address: hirasaka@pusan.ac.kr

Weakly Locally Projective Graphs and Groups

Alexander Ivanov

I will discuss graphs Γ with edge-transitive automorphism group G, which are (a) bipartite with vertices in one part having valency $\alpha \in \{2, 3\}$ (they are called *lines*), and the vertices in the other part having valency $2^n - 1$ for some $n \geq 2$ (they are called *points*); (b) the stabilizer in G of a point induces the natural doubly transitive action of $L_n(2)$ on the set of lines containing this point. The ultimate goal is to classify the amalgams $\{G(p), G(l)\}$, where $\{p, l\}$ is an incident point-line pair. The problem was solved by D.Djoković and G.Miller in 1980 for $(\alpha, n) = (2, 2)$ based on earlier work by W.Tutte (1947) and C.Sims (1968); by D.Goldschmidt in 1980 for $(\alpha, n) = (3, 2)$, by S.Shpectorov and the author in 2002 for $\alpha = 2$ and arbitrary n based on results due to V.I.Trofomov (published in a series of papers through 1990's). The classical examples for $\alpha = 3$ are point-line collinearity graphs of the linear and symplectic dual polar spaces over $GF(2)$; while the exceptional ones are the Cooperstain geometry of $G_2(3)$ and the tilde geometries of M_{24}, He, Co_1 and the Monster group.

Imperial College London, United Kingdom

Permutation Modules for the Symmetric Groups

Mario Mainardis

I shall present a method for computing the irreducible constituents of the permutation characters associated to subgroups of S_n normalising a given partition of $\{1, \ldots, n\}$. This is part of a joint project with C. Franchi and A. A. Ivanov for determining the Majorana representations of the symmetric groups.

Università di Udine, Italy
E-mail address: mario.mainardis@uniud.it

Butson-Hadamard matrices in association schemes of class 6 on Galois rings of characteristic 4

Akihiro Munemasa

A complex Hadamard matrix is a square matrix W of order n which satisfies $W\overline{W}^\top = nI$ and all of whose entries are complex numbers of absolute value 1. A complex Hadamard matrix is said to be Butson-type, if all of its entries are roots of unity. In an earlier work [2], we proposed a method to classify symmetric complex Hadamard matrices belonging to the Bose–Mesner algebra of a symmetric association scheme.

Galois rings have been used to construct certain association schemes (see [3, 4, 5]), and certain properties of association schemes obtained from Galois rings have been investigated in [1].

In this talk, we give a construction of a nonsymmetric association scheme $\mathfrak{X}$ of class 6 on the Galois ring of characteristic 4, and classify hermitian complex Hadamard matrices belonging to the Bose–Mesner algebra of $\mathfrak{X}$. We show that such a matrix is necessarily a Butson-type matrix whose entries are 4-th roots of unity. One of the family of such matrices actually belongs to the smaller Bose–Mesner algebra of a class 3 fusion scheme. These fusion schemes are the only family (parametrically) of class 3 nonsymmetric schemes whose Bose–Mesner algebra contains a (non-real) hermitian complex Hadamard matrix.

This is based on joint work with Takuya Ikuta.

References

[1] S. Evdokimov and I. Ponomarenko, *Normal cyclotomic schemes over a finite commutative ring*, St. Petersburg Math. J. **19** (2008), no. 6, 911–929.

[2] T. Ikuta and A. Munemasa, *Complex Hadamard matrices contained in a Bose–Mesner Algebra*, Spec. Matrices **3** (2015), 91–110.

[3] T. Ito, A. Munemasa, and M. Yamada, *Amorphous association schemes over the Galois rings of characteristic* 4, European J. Combin. **12** (1991), 513–526.

[4] J. Ma, *Three-class association schemes on Galois rings in characteristic* 4, Graphs and Combinatorics **23** (2007), no. 1, 73–86.

[5] M. Yamada, *Distance-regular digraphs of girth* 4 *over an extension ring of* $\mathbb{Z}/4\mathbb{Z}$, Graphs and Combinatorics **6** (1990), no. 4, 381–394.

Tohoku University, Japan

E-mail address: munemasa@math.is.tohoku.ac.jp

On non-commutative association schemes of rank 6

Mikhail Muzychuk

An association scheme is a coloring of a complete graph satisfying certain regularity conditions. It is a generaliztion of groups and has many applications in algebraic combinatorics. Every association scheme yields a special matrix algebra called the Bose-Mesner algebra of a scheme. A scheme is called commutative if its Bose-Mesner algebra is commutative. Commutative schemes were the main topic of the research in this area for decades. Only recently non-commutative association schemes attracted attention of the researches. In my talk I'll present the results about non-commutative association schemes of the smallest possible rank, namely the rank 6. This is a joint work with A. Herman and B. Xu.

Department of Computer Sciences, Netanya Academic College, Israel
E-mail address: muzy@netanya.ac.il

Constructing the automorphism group of a finite p-group

Eamonn O'Brien

Constructing the automorphism group of a finite group remains challenging. The critical hard case is that of a finite p-group P where much effort has been invested over the past 20 years in developing recursive algorithms which work down a central series for P. If we can locate characteristic structure in P, then we can often readily solve the problem.

The real challenge remains class 2 p-groups of exponent p. Here we can readily reduce to the following concrete problem. Let $G := \mathrm{GL}(d, \mathrm{GF}(p))$ act on the exterior square representation U of the natural module; given an explicit subspace W of U construct its stabiliser in G.

In this talk we will survey the problem and outline algorithmic approaches. In particular we will report on related work to uncover hidden characteristic structure and recent joint work with Brooksbank and Wilson on graded algebras. Both offer new hope of progress on this intractable problem.

University of Auckland, New Zealand

Fusion systems defined on p-groups with p an odd prime

Chris Parker

Given a prime p, a group G and Sylow p-subgroup S of G, the saturated fusion systems determined by G on S captures all the information about conjugacy of subgroups and elements of S in G. In general a saturated fusion system is a category which encapsulates all the properties one would expect if the fusion system came from a group as just described. However there exist saturated fusion systems which do not come from groups. Such systems are called exotic fusion systems. For odd primes p, there are many families of exotic fusion systems. In this talk, I will describe some of these families and also present results whose aim is to understand how these fusion systems come about.

University of Birmingham, United Kingdom
E-mail address: c.w.parker@bham.ac.uk

On the base sizes of linear groups

Károly Podoski

A linear group G acting on a finite space V is called coprime linear group if $(|G|, |V|) = 1$. Based on our earlier result that is every coprime primitive linear groups admits a base size two, László Pyber asked whether there exists a positive integer constant c, for coprime linear groups, such that the probability of a random c-tuple in V is a base for G tends to 1 as $|G| \to \infty$. We answered this question affirmatively for solvable linear groups by showing that we can choose $c = 9$ if the group is coprime and $c = 13$ if the group is not coprime. This is a joint work with Zoltán Halasi.

Alfréd Rényi Institute of Mathematics, Hungary
E-mail address: podoski.karoly@renyi.mta.hu

Regular subgroups of permutation groups: estimates, algorithms, and applications

Ilia Ponomarenko

A transitive permutation group is said to be *regular* if the one-point stabilizer of it is trivial. Regular subgroups of permutation groups arise in many natural contexts, for example, in group factorizations [4], Schur rings [7], Cayley graphs [1], etc. Given a group H and a permutation group K, we are interested in the set

$$\mathrm{Reg}(K, H) = \{G \leq K : G \text{ is regular and } G \cong H\}.$$

A set $B \subset \mathrm{Reg}(K, H)$ containing exactly one element in each orbit of K acting on $\mathrm{Reg}(K, H)$ by conjugation, is called an H*-base* of K. The cardinality of an H-base of K is denoted by $b_H(K)$.

Using terminology and arguments of [1], one can see that if K is the automorphism group of an object of a concrete category $\mathscr{C}$, then B yields a full set of pairwise non-equivalent representations of this object as a Cayley object over H in $\mathscr{C}$. As $\mathscr{C}$, one can take, for example, the category of finite graphs or other combinatorial structures.

Let $H = C_n$ be a cyclic group of order n. Then, obviously, $b_H(K) \leq c(K)$, where $c(K)$ is the number of the conjugacy classes of full cycles contained in K. It was proved in [5] that the latter number does not exceed n.[1] Thus, in this case $b_H(K) \leq n$. The first main result is given by the following statement proved in [6].

Theorem 1. *A C_n-base of a permutation group of degree n can be constructed in time* $\mathrm{poly}(n)$.

Remark. Theorem 1 generalizes a result in [2], where an efficient algorithm for finding a C_n-base of K was constructed in the case where the group K is solvable or 2-closed, i.e., the automorphism group of a graph.

It can be shown that if p is a prime and $H = C_p \times C_p$, then $b_H(K) \leq n$. On the other hand, the following result was recently proved in [3].

Theorem 2. *For every prime p, there exists a p-group $K \leq \mathrm{Sym}(p^3)$ such that $b_H(\Gamma) \geq p^{p-2}$, where $H = C_p \times C_p \times C_p$.*

The groups K in Theorem 2 are not 2-closed. It would be interesting to find 2-closed groups K for which the inequality on Theorem 2 holds true.

References

[1] L. Babai, *Isomorphism problem for a class of point symmetric structures*, Acta Math. Acad. Sci. Hung. **29** (1977), no. 3, 329–336.

[2] S. Evdokimov and I. Ponomarenko, *Recognizing and isomorphism testing circulant graphs in polynomial time*, Algebra and Analysis **15** (2003), no. 6, 1–34.

[3] S. Evdokimov, M. Muzychuk, and I. Ponomarenko, *A family of permutation groups with exponentially many non-conjugated regular elementary abelian subgroups*, arXiv:1609.08467 (2016), 1–6.

[1] More exactly, under the Classification of Finite Simple Groups, $c(K) \leq \varphi(n)$ where φ is the Euler function, ibid.

[4] M. W. Liebeck, C. Praeger, and J. Saxl, *Regular subgroups of primitive permutation groups*, Memoirs Amer. Math. Soc. **203** (2010), no. 952, 1–88.

[5] M. Muzychuk, *On the isomorphism problem for cyclic combinatorial objects*, Discr. Math. **197/198** (1999), 589–606.

[6] M. Muzychuk and I. Ponomarenko, *Finding a cycle base of a permutation group*, arXiv:1702.05292 (2017), 1–14.

[7] H. Wielandt, *Finite permutation groups*, Academic press, New York - London (1964).

St.Petersburg Department of V.A.Steklov Institute of Mathematics, of the Russian Academy of Sciences, Russia

E-mail address: `inp@pdmi.ras.ru`

How to avoid the Classification Theorem of Finite Simple Groups in Asymptotic Group Theory

László Pyber

The Classification of Finite Simple Groups (CFSG) is a monumental achievement and a seemingly indispensable tool in modern finite group theory. By now there are a few results which can be used to bypass this tool in a number of cases, most notably a theorem of Larsen and Pink which describes the structure of finite linear groups of bounded dimension over finite fields.

In a few cases more ad hoc arguments can be used to delete the use of CFSG from the proofs of significant results. The talk will among others discuss a recent example due to the speaker: how to obtain a CFSG-free version of Babai's quasipolynomial Graph Isomorphism algorithm by proving a Weird Lemma about permutation groups.

Alfréd Rényi Institute of Mathematics, Hungary

Half-axes in power associative algebras

Yoav Segev

Let A be a commutative, non-associative algebra over a field F of characteristic $\neq 2$. A half-axis in A is an idempotent $e \in A$ such that e satisfies the Peirce multiplication rules in a Jordan algebra, and, in addition, the 1-eigenspace of ad_e (multiplication by e) is one dimensional.

In this talk we show that if A is power associative, then one gets (very) interesting identities between elements in the eigenspaces of ad_e. We use these identities to prove that if A is a primitive axial algebra of Jordan type half (i.e., A is generated by half-axes), and $|F| > 3$, then A is a Jordan algebra.

Ben-Gurion university, Israel
E-mail address: `yoavs@math.bgu.ac.il`

Computing axial algebras for concrete groups

Sergey Shpectorov

The paradigm of axial algebras is a broad generalization of the axioms of Majorana algebras of Ivanov. In the first part of the talk we will present the basics of axial algebras (fusion rules, axes, Frobenius form), as introduced by Hall, Rehren and the speaker, and explain how a grading on the fusion rules leads to automorphisms of the algebra. We will also introduce main motivating examples: the Griess-Norton algebra for the Monster group, Jordan algebras and Matsuo algebras.

Since axial algebras are related to groups, it is natural to ask whether it is possible to determine axial algebras for a concrete group G. All algebras for G fall into cases specified by shape. We will discuss this concept, initially introduced by Ivanov for Majorana algebras, and then turn to specific classes of axial algebras.

First we will consider computing axial algebras of Jordan type for concrete groups. This is a relatively simple case due to the specific features of this class of algebras: every algebra is linearly spanned by axes and it admits a Frobenius form. This leads to a pretty straightforward algorithm, used by McInroy and the speaker to compute algebras of Jordan type for a number of finite groups, with the largest example being the sporadic group J_2.

After that we will turn to the more complicated class of M-algebras, which are the axial version of Majorana algebras. We first review the contribution of Seress, who constructed a great many 2-closed algebras, including some big ones. We will then present a theoretical construction allowing to overcome the 2-closeness restriction, as well as its computational counterpart based on the concepts of partial algebra and algebra expansion. We will illustrate this by the computation of algebras for twelve shapes of the group S_4 and some further groups.

We will conclude with discussing the requirements on the linear algebra algorithms underlying our approach.

University of Birmingham, United Kingdom
E-mail address: s.shpectorov@bham.ac.uk

The classification of the oriented regular representations

Pablo Spiga

Broadly speaking, this talk is divided into three parts related to graph representations of groups. First we aim to give a complete answer to a 1980 question of Lazslo Babai: "Which [finite] groups admit an oriented graph as a DRR?" That is, which finite groups admit an oriented regular representation (ORR)? In the second part of this talk, we address another widely open question of Babai: asymptotic enumeration of Cayley (di)graphs. In the last part of this talk, we address a very recent question of Watkins and Tucker on Frobenius regular representations of finite groups.

University of Milano-Bicocca, Italy
E-mail address: pablo.spiga@unimib.it

On isomorphism problem for coherent configurations associated with nonsolvable groups

Andrey Vasil′ev

A coherent configuration $\mathcal{X} = (\Omega, S)$ on a set Ω is said to be schurian if the set S of its basis relations is precisely the set $\mathrm{Orb}_2(G, \Omega)$ of 2-orbits of some permutation group $G \leq \mathrm{Sym}(\Omega)$, that is the set of the orbits of the naturally induced action of G on $\Omega \times \Omega$. Clearly, in the case of a schurian configuration $\mathcal{X}$, this group G is a subgroup of the automorphism group $\mathrm{Aut}(\mathcal{X})$. So starting with G and trying to find $\mathrm{Aut}(\mathcal{X})$ or the set $\mathrm{Iso}(\mathcal{X}, \mathcal{X}')$ of all isomorphisms from $\mathcal{X}$ to an arbitrary coherent configuration $\mathcal{X}'$, we have an advantage knowing some predetermined information about $\mathrm{Aut}(\mathcal{X})$. The same picture arises in the case of Cayley graphs (or Cayley schemes, which are coherent configurations as well). Indeed, if $\Gamma = \mathrm{Cay}(G, X)$ is the Cayley graph for a group G with a connection set X, then G is included as a regular subgroup in $\mathrm{Aut}(\Gamma)$. We are going to discuss some new results and techniques on the isomorphism problem for combinatorial objects associated with a group G in the described way, concentrating on the cases when G is a nonabelian simple or almost simple group.

Sobolev Institute of Mathematics and Novosibirsk State University, Russia
E-mail address: vasand@math.nsc.ru

Number of Sylow subgroups in finite groups

Evgeny Vdovin

For a finite group G, denote by $\nu_p(G)$ the number of Sylow p-subgroups of G. It is a trivial exercise to check that for every subgroup H of G, the inequality $\nu_p(H) \leq \nu_p(G)$ holds. However $\nu_p(H)$ does not divide $\nu_p(G)$ in general. In 2003 G. Navarro proved that $\nu_p(H)$ divides $\nu_p(G)$ for every $H \leq G$ if G is p-solvable. We prove that $\nu_p(H)$ divides $\nu_p(G)$ for every $H \leq G$ if this property holds for every nonabelian composition factor of G. Thus we obtain a substantial generalization of Navarro's result and also give an alternative proof for Navarro's result.

We say that a group G satisfies $\mathbf{DivSyl}(p)$ if $\nu_p(H)$ divides $\nu_p(G)$ for every $H \leq G$.

Theorem. *Let*

$$1 = G_0 < G_1 < \ldots < G_n = G$$

be a refinement of a chief series of G. Assume that for each nonabelian factor G_i/G_{i-1} and for every p-subgroup P of $\mathrm{Aut}_G(G_i/G_{i-1})$, the group $P(G_i/G_{i-1})$ satisfies $\mathbf{DivSyl}(p)$. Then G satisfies $\mathbf{DivSyl}(p)$.

References

[1] G. Navarro, *Number of Sylow subgroups in p-solvable groups*, Proc. Amer. Math. Soc. **131** (2003), no. 10, 3019–3020.

Sobolev Institute of Mathematics, Russia

E-mail address: vdovin@math.nsc.ru

The Exchange Condition for Hypergroups

Paul-Hermann Zieschang

Let S be a set, and let μ be a map from $S \times S$ to the power set of S. For any two elements p and q of S, we write pq instead of $\mu(p, q)$ and assume that pq is not empty. For any two non-empty subsets P and Q of S, we define the *complex product* PQ to be the union of the sets pq with $p \in P$ and $q \in Q$. If one of the two factors in a complex product consists of a single element, say s, we write s instead of $\{s\}$ in that product.

Following (and generalizing) Frédéric Marty's terminology [1] we call S a *hypergroup* (with respect to μ) if the following three conditions hold.

1. $\forall\ p, q, r \in S$: $p(qr) = (pq)r$.
2. $\exists\ e \in S\ \forall\ s \in S$: $se = \{s\}$.
3. $\forall\ s \in S\ \exists\ s^* \in S\ \forall\ p, q, r \in S$: $p \in q^*r^* \Rightarrow q \in r^*p^*$ and $r \in p^*q^*$.

Each association scheme satisfies the above three conditions with respect to its complex multiplication; cf. [2; Lemma 1.3.1, Lemma 1.3.3(ii), Lemma 1.3.3(i)]. Thus, hypergroups generalize association schemes.

I will explain how association schemes may take advantage of the structure theory of hypergroups. Special attention will be given to the embedding of the theory of buildings as well as the theory of twin buildings into scheme theory; cf. [3].

References

[1] F. Marty, *Sur une généralisation de la notion de groupe*, Huitième Congres des Mathématiciens (1934), 45–59.

[2] P.-H. Zieschang, *Theory of Association Schemes*, Springer Monographs in Mathematics (2005).

[3] P.-H. Zieschang, *Hypergroups*, Max-Planck-Institut für Mathematik Preprint Series **97** (2010).

University of Texas Rio Grande Valley, USA

E-mail address: zieschang@utrgv.edu

Contributed talks

Intersection of conjugate solvable subgroups in finite groups

Anton Baykalov

Assume that a finite group G acts on a set Ω. An element $x \in \Omega$ is called a *regular point* if $|xG| = |G|$, i.e. if the stabilizer of x is trivial. Define the action of the group G on Ω^k by

$$g : (i_1, \ldots, i_k) \mapsto (i_1 g, \ldots, i_k g).$$

If G acts faithfully and transitively on Ω, then the minimal number k such that the set Ω^k contains a G-regular point is called the *base size* of G and is denoted by $b(G)$. For a positive integer m the number of G-regular orbits on Ω^m is denoted by $Reg(G, m)$ (this number equals 0 if $m < b(G)$). If H is a subgroup of G and G acts by the right multiplication on the set Ω of right cosets of H then G/H_G acts faithfully and transitively on the set Ω. (Here $H_G = \cap_{g \in G} H^g$.) In this case we denote $b(G/H_G)$ and $Reg(G/H_G, m)$ by $b_H(G)$ and $Reg_H(G, m)$ respectively.

Thus $b_H(G)$ is the minimal number k such that there exist $x_1, \ldots, x_k \in G$ with $H^{x_1} \cap \ldots \cap H^{x_k} = H_G$.

Consider Problem 17.41 b) from "Kourovka notebook" [1]:

Let H be a solvable subgroup of finite group G that has no nontrivial solvable normal subgroups. Do there always exist five conjugates of H whose intersection is trivial?

The problem is reduced to the case when G is almost simple in [2]. Specifically, it is proved that if for each almost simple group G and solvable subgroup H of G inequality $Reg_H(G, 5) \geq 5$ holds then for each finite nonsolvable group G and maximal solvable subgroup H of G inequality $Reg_H(G, 5) \geq 5$ holds.

In the talk we discuss the recent progress in the solution of the problem.

References

[1] *The Kourovka Notebook: Unsolved problems in group theory*, 18 ed., arXiv:1401.0300.

[2] E. P. Vdovin, *On the base size of a transitive group with solvable point stabilizer*, J. Algebra Appl. **11** (2012), no. 1, 1250015 (14 pages)

Novosibirsk State University, Russia

E-mail address: `anton188@bk.ru`

On 2-closures of abelian groups

Dmitry Churikov

The goal of this talk is to discuss how to determine whether a finite abelian group is 2-closed. It is known that the 2-closure of a finite nilpotent (so abelian as well) permutation group of degree n can be constructed in time poly(n) [1]. However, it would be interesting to have an explicit criterion of 2-closedness. In [2], such a criterion was introduced as a corollary of the main result. Unfortunately this criterion is wrong and we give a counterexample. Furthermore, we introduce our variant of a criterion.

References

[1] I. Ponomarenko, *Graph isomorphism problem and 2-closed permutation groups*, Applicable Algebra in Engineering, Communication and Computing **5** (1994), no. 1, 9–22.

[2] A. Zelikovskiĭ, *The König problem for abelian permutation groups*, Vestsi Akad. Navuk BSSR Ser. Fiz.-Mat. Navuk **125** (1989), no. 5, 34–39.

Novosibirsk State University, Russia
E-mail address: churikovdv@gmail.com

Separability of Cayley schemes over abelian p-groups

Grigory Ryabov

A Cayley scheme is called *separable* with respect to the class of Cayley schemes $\mathcal{K}$ if it is determined up to isomorphism in $\mathcal{K}$ only by its intersection numbers. We say that an abelian group G is *separable* if every Cayley scheme over G is separable with respect to the class of Cayley schemes over abelian groups. Denote the cyclic group of order n by C_n. Let G be a noncyclic abelian p-group. From the previously known results it follows that if G is separable then G is isomorphic to $C_p \times C_{p^k}$ or $C_p \times C_p \times C_{p^k}$, where $p \in \{2, 3\}$ and $k \geq 1$. In fact, all Cayley schemes over $C_p \times C_{p^k}$ were classified in [1] for $p = 2$ and in [2] for $p = 3$. By using this classification we prove that the groups $G = C_p \times C_{p^k}$ are separable whenever $p \in \{2, 3\}$. The obtained result implies the solution of the graph isomorphism problem in time $|G|^{O(1)}$ in the class of graphs that isomorphic to Cayley graphs over G. Also based on the description of all Cayley schemes over G we solve in time $|G|^{O(1)}$ the following problem: given a graph Γ on $|G|$ verticies determine whether Γ is isomorphic to a Cayley graph over G.

References

[1] M. Muzychuk, I. Ponomarenko, *On Schur 2-groups*, Zapiski Nauchnykh Seminarov POMI **435** (2015), 113-162.

[2] G. Ryabov, *On Schur p-groups of odd order*, J. Algebra Appl. **16** (2017), no. 3, 1750045 (29 pages).

Novosibirsk State University, Russia

E-mail address: gric2ryabov@gmail.com

On hypocritical groups

Saveliy Skresanov

A variety of groups is *locally finite* if its finitely generated groups are finite. L. F. Harris [1] introduced the following definition: a group D is called *hypocritical*, if whenever D is in a locally finite variety generated by a set of groups $\mathfrak{X}$, then D is a section of a group from $\mathfrak{X}$.

In this talk groups which are an extension of a p-group by a p'-group are considered from the standpoint of hypocriticality. We discuss known facts in the case of an abelian Sylow p-subgroup and state the corresponding result in the case of an extraspecial Sylow p-subgroup of order p^3.

References

[1] L. F. Harris, *Hypocritical and sincere groups*, Proc. Second Internat. Conf. Theory of Groups **372** (1973), 333-336.

Novosibirsk State Univeristy, Russia
E-mail address: s.skresanov@g.nsu.ru