ASYMPTOTIC OF POTENTIALS OF INCLUSIONS
IN HIGH-CONTRAST HIGH-FILLED MEDIUM

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Denote $P = [-1, 1]^n \subset \mathbb{R}^n$ ($n=2, 3$) domain containing a system of not torching domains $\{D_i, i = 1, \ldots, N\}$ called inclusions and consider the following boundary value problem

$$\Delta \varphi = 0 \quad \text{in} \quad Q; \quad \varphi(x) = t_i \quad \text{in} \quad D_i, \quad i = 1, \ldots, N; \quad \int_{\partial D_i} v_n \, dx = 0, \quad i = 1, \ldots, N; \quad \frac{\partial \varphi}{\partial n}(x) = 0 \quad \text{on} \quad \partial Q_{\text{lat}}; \quad \varphi(x) = -1 \quad \text{on} \quad \partial D^-, \quad \varphi(x) = 1 \quad \text{on} \quad \partial D^+;$$

(1)

$\partial D^-$ and $\partial D^+$ mean the top and bottom boundaries of $P$, $\partial Q_{\text{lat}}$ mean the right and left boundaries of $P$.

The unknowns in (1) are the function $\varphi(x)$ in domain $Q$ and numbers $\{t_i, i = 1, \ldots, N\}$ (potentials of inclusions $\{D_i, i = 1, \ldots, N\}$).

The problem goes back to works by Maxwell and Rayleigh. In 1960–2000, it was investigated asymptotic behavior of the problem under condition that characteristic distance $\delta$ between inclusions is small [1]. The modern stage of the analysis of the problem is related to the method of network approximation for high-contrast boundary value problems [2].

The network approximation for the problem (1) is constructed under assumption the inclusion interacts with its neighbors only and flux between $i$-th and $j$-th inclusions is equal to $C_{ij}^{(2)}(t_i - t_j)$, where $C_{ij}^{(2)}$ is capacity of these two inclusions $\mathbb{R}^n$. The network approximation has the form

$$\sum_{j \in N_i} C_{ij}^{(2)}(t_i - t_j) = 0, \quad t_i = \pm 1 \quad \text{on} \quad S^\pm,$$

(2)

where $S^+$ and $S^-$ are indices of quasi inclusions [3, 4] (portions of top and bottom boundaries corresponding to the near-boundary inclusions).

Let us denote solution of the boundary value problem (2) by $\{t_i^{\text{net}}, i = 1, \ldots, N\}$.

The object of interest was asymptotic of total flux (it is equal to effective conductivity, total energy or capacity of the system of inclusions) as $\delta \to 0$.

The complete proof of asymptotic equivalentness of the total flux corresponding to the problems (1) and (2) as $\delta \to 0$ for disordered (not periodic) inclusions was given not long ago ([3] for a system of planar disks and [4] for general case). It was found that the phenomenon of the network approximation is not an independent problem but a special case of I. E. Tamm asymptotic shielding effect [5, 6].

In [7] (first, to the best knowledge of the author) it was formulated the problem of relationship of potentials $\{t_i, i = 1, \ldots, N\}$ and $\{t_i^{\text{net}}, i = 1, \ldots, N\}$ of inclusions determined from the boundary value problem (1) and network problem (2) and it was proved convergence of the potentials of inclusion as $\delta \to 0$ for the the inclusions in the shape of planar circular disks.

In the present paper it is proved that $|t_i - t_i^{\text{net}}| \to 0, i = 1, \ldots, N$ as $\delta \to 0$ for the inclusions, which shapes satisfy the condition of existence of I. E. Tamm asymptotic shielding effect (in particular, for inclusions with smooth boundaries) [5].

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REFERENCES


