HOMOGENIZED EQUATIONS FOR SHORT-TIME FILTRATION PROCESSES AND ACOUSTIC WAVE PROPAGATION IN ELASTIC POROUS MEDIA

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We consider a problem of a joint motion of a deformable solid, perforated by a system of channels (pores) and a fluid occupying a porous space. In dimensionless variables differential equations of the problem for the dimensionless displacement vector $\mathbf{w}^\varepsilon$ of the continuum medium have a form:

$$
\alpha^\varepsilon \rho^\varepsilon \frac{\partial^2 \mathbf{w}^\varepsilon}{\partial t^2} = \text{div}_x \{ \chi^\varepsilon \alpha_\mu \mathbf{D}(x, \frac{\partial \mathbf{w}^\varepsilon}{\partial t}) + (1 - \chi^\varepsilon) \alpha_\lambda \mathbf{D}(x, \mathbf{w}^\varepsilon) - (q^\varepsilon + \pi^\varepsilon) \mathbb{I} \} + \rho^\varepsilon \mathbf{F},$$

$$
q^\varepsilon = -\chi^\varepsilon \alpha^\varepsilon \text{div}_x \mathbf{w}^\varepsilon, \quad \pi^\varepsilon + (1 - \chi^\varepsilon) \alpha_\eta \text{div}_x \mathbf{w}^\varepsilon = 0.
$$

In this model the characteristic function of the porous space $\chi^\varepsilon$, coefficient $\rho^\varepsilon = \rho_f \chi^\varepsilon + \rho_s (1 - \chi^\varepsilon)$ and a dimensionless vector $\mathbf{F}(x,t)$ of distributed mass forces are known functions. Dimensionless parameters $\alpha_i$ ($i = \tau, \nu, \ldots$) depend on the small parameter $\varepsilon = l/L$, where $l$ is a characteristic size of pores and $L$ is a characteristic size of the entire porous body. Although the problem is linear, it is very hard to tackle due to the fact that its main differential equations involve non-smooth oscillatory coefficients under the differentiation operators. We suggest the rigorous justification for homogenization procedures as $\varepsilon$ tends to zero, while the porous body is geometrically periodic and a characteristic time of processes is small enough. Such kind of models may describe, for example, hydraulic fracturing or acoustic wave propagation. As the results we derive different types of homogenized equations involving non-isotropic Stokes system for fluid velocity coupled with acoustic equations for the solid component, or non-isotropic Stokes system for one-velocity continuum media, or different types of acoustic equations for one- or two-velocity continuum media, depending on ratios between physical parameters. The proofs are based on Nguetseng’s two-scale convergence method of homogenization in periodic structures.