ONE JUMP AWAY: SPECTRA OF DIFFERENTIALLY CLOSED FIELDS

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The spectrum $\mathrm{Spec}(\mathcal{A})$ of a countable structure $\mathcal{A}$ is the set of all Turing degrees of isomorphic copies of $\mathcal{A}$. It is well known that every possible spectrum of a first-order structure in a finite signature is also the spectrum of a symmetric irreflexive graph. In this sense, the theory of such graphs is complete. Other complete theories include those of partial orders, lattices, 2-step nilpotent groups, and fields; whereas more restrictive theories such as linear orders, Boolean algebras, and algebraically closed fields are known to be incomplete.

The theory $\mathbf{DCF}_0$ of differentially closed fields of characteristic 0 has become prominent in model theory: it is the analogue of algebraically closed fields when the signature also includes a differential operator on the field elements. We will present joint work between David Marker and the speaker, showing that this theory is incomplete for spectra, in the sense given above. However, it comes close to completeness. The spectra of models of $\mathbf{DCF}_0$ are exactly those sets of the form

$$\{\boldsymbol{d} \ : \ \boldsymbol{d}' \in \mathrm{Spec}(G)\},$$

for all countable graphs G. So this theory may be considered to be precisely one jump away from completeness. The crux of the equality, in one direction, is a demonstration that every model of $\mathbf{DCF}_0$ of low degree has a computable copy – a property well-known for Boolean algebras (although the details are different) and recently studied in more generality by several researchers as "strong jump inversion."

Bibliography

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