REIDEMEISTER SPECTRUM OF CLASSICAL LINEAR GROUPS

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Let G be a group and φ be an automorphism of G. Two elements x, y of G are said to be (twisted) φ-conjugated if there exists an element $z \in G$ such that $x = zy\varphi(z)^{-1}$. The relation of φ-conjugation is an equivalence relation on G and it divides the group into φ-conjugacy classes. The number $R(\varphi) \in \mathbb{N} \cup \{\infty\}$ of these classes is called the Reidemeister number of the automorphism φ.

The Reidemeister spectrum $\mathrm{Spec}_R(G)$ of a group G is the subset of $\mathbb{N} \cup \{\infty\}$ of the form $\mathrm{Spec}_R(G) = \{R(\varphi) \mid \varphi \in \mathrm{Aut}(G)\}$. If $\mathrm{Spec}_R(G) = \{\infty\}$, then the group G is said to possess the R_∞-property. The problem of classifying groups which possess the R_∞-property was proposed by A. Fel'shtyn and R. Hill in [1]. The study of this problem has been quite an active research topic in recent years.

Duirng the talk, at first, we are going to discuss several applications of twisted conjugacy relations and the R_∞-property in different areas of mathematics, and then talk about twisted conjugacy classes, Reidemeister spectrum and the R_∞-property for classical linear groups. Most of the results we are going to discuss are collected in papers [3, 4, 5].

If $\mathbb{F}$ is an algebraically closed field of zero characteristic such that the transcendence degree of $\mathbb{F}$ over $\mathbb{Q}$ is finite, then a lot of linear algebraic groups are known to possess the R_∞-property (see, for example, [2]). In the talk we are going, in particular, to consider linear algebraic groups over an algebracally closed field $\mathbb{F}$ of zero characteristic in the case when the transcendence degree of $\mathbb{F}$ over $\mathbb{Q}$ is infinite. It turns out that a lot of linear groups (including Chevalley groups of classical series) over such fields do not possess the R_∞-property.

References

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