CODING AND DECODING IN CLASSES OF STRUCTURES

ALEXANDRA SOSKOVA

Abstract

Friedman and Stanley [1] introduced Borel embeddings as a way of comparing classification problems for different classes of structures. A Borel embedding represents a uniform procedure for coding structures from one class in structures from another. Many Borel embeddings are actually Turing computable.When a structure $\mathcal{A}$ is coded in a structure $\mathcal{B}$, the effective decoding is represented by a uniform effective interpretation [2]. Part of the effective interpretation is Medvedev reduction.

The class of undirected graphs and the class of linear orderings both lie on top under Turing computable embeddings. The standard Turing computable embeddings of directed graphs (or structures for an arbitrary computable relational language) in undirected graphs come with uniform effective interpretations. We give examples of graphs that are not Medvedev reducible to any linear ordering, or to the jump of any linear ordering. Any graph can be Medvedev reducible to a linear ordering using computable Σ_3 formulas. Friedman and Stanley [1] gave a Turing computable embedding L of directed graphs in linear orderings. We show that there do not exist $L_{\omega_1\omega}$-formulas that uniformly interpret the input graph G in the output linear ordering $L(G)$. This is joint work with J. Knight, and S. Vatev [3].

We have also a positive result — we prove that the class of fields are effectively interpreted in the class of Heisenberg groups generalising an old Maltsev's result. The second part is a joint work with R. Alvir, W. Calvert, G. Goodman, V. Harizanov, J. Knight, R. Miller, A. Morozov, and R. Weisshaar [4].

References

[1] H. Friedman and L. Stanley, "A Borel reducibility theory for classes of countable structures", *JSL*, vol. 54(1989), pp. 894–914.

[2] M. Harrison-Trainor, A. Melnikov, R. Miller, and A. Montalbán, "Computable functors and effective interpretability", *JSL*, vol. 82(2017), pp. 77–97.

[3] J. Knight, A. Soskova, S. Vatev "Coding in graphs and linear orderings", *JSL*, vol. 85(2)(2020), pp. 673–990.

[4] R. Alvir, W. Calvert, G. Goodman, V. Harizanov, J. Knight, R. Miller, A. Morozov, A. Soskova, and R. Weisshaar, "Interpreting a field in its Heisenberg group", to appear in *JSL*.

Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski" 5, J. Bourchier Blvd, 1164 Sofia, Bulgaria

Email address: asoskova@fmi.uni-sofia.bg

The research was supported by FNI-SU 80-10-136/26.03.2021 and NSF Grant DMS -1600625.