Problems on computable numberings

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Let $\mathbb{K}$ be a class of subsets of ω that is closed under the Cartesian products. Informally, we can consider a computable numbering of a family $\mathfrak{F} \subseteq \mathbb{K}$ as a sequence of elements of $\mathfrak{F}$ that is uniformly enumerable in $\mathbb{K}$. More formally, we say that a surjective mapping $\nu : \omega \mapsto \mathfrak{F}$ is a *computable numbering* if

$$\{< x, n >: x \in \nu(n)\} \in \mathbb{K},$$

and that $\mathfrak{F}$ is a *computable family* if it possesses a computable numbering.

We consider only the pairs $\langle \mathbb{K}, \mathfrak{F} \rangle$ so that and $\mathfrak{F}$ is a computable family of sets from the class $\mathbb{K}$. The general notion for reducibility of numberings is presupposed to use in the talk.

$\mathcal{R}(\mathfrak{F})$ stands for the Rogers semilattice of computable numberings of $\mathfrak{F}$. The problems on computable numberings are usually formulated in terms of Rogers semilattices. Our goal is to discuss the current state of study the Rogers semilattices for classes $\mathbb{K}$ of

- computably enumerable sets,
- the sets of a given level of the arithmetical hierarchy,
- the sets of a finite or an infinite level of the Ershov hierarchy,
- the sets of a low levels of the analytical hierarchy.

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