MODAL EXTENSIONS OF BELNAP-DUNN LOGIC

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We survey a number of recent results concerning a certain family of four-valued modal logics. The foundation of this family is a four-valued logic developed by Nuel Belnap and Michael Dunn. This logic is motivated by the consideration of a computer that receives information on truth and falsity of statements from different sources. This way the computer might receive information that a certain fact is true, false, both true and false or neither true nor false, which gives one epistemically motivated truth values. Defining conjunction $\wedge$, disjunction $\vee$ and negation $\sim$ on threse four values one obtains a logic which technically coincides with the logic known as *first-degree entailment* FDE.

To reflect this connection the family of modal logics we are interested in was designated as *FDE-based modal logics* by Sergei Odintsov and Heinrich Wansing. This designation might be too broad for the following reason: all systems we will discuss are not contrapositive in the sense that $\varphi \vdash \psi$ does not necessarily imply $\sim \psi \vdash \sim \varphi$. The leads to perhaps the most interesting feature of the logics in question: they all lack the usual replacement theorem. What they have instead is the so-called weak replacement property: one can replace φ with ψ as long as φ is equivalent to ψ and $\sim \varphi$ is equivalent to $\sim \psi$ in the usual sense. On the other hand, FDE itself is contrapositive in this sense, although there is an axiom system for the logic in which the contraposition rule is merely admissible.

Some of the systems we will discuss are Belnapian modal logics of Odintsov and Wansing, modal bilattice logic MBL developed by Umberto Rivieccio, Achim Jung and Ramon Jansana, Graham Priest's $\mathsf{K}_{\mathsf{FDE}}$, Lou Goble's $\mathsf{KN4}$ and a few more. Most of the works outlined will be dedicated to highlighting connections between these logics through the lenses of proof theory (including Hilbert-style systems, FDE-style systems and display calculi), relational semantics and algebras (including twist-structures and residuated lattices).

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