

# THE APPROXIMABILITY OF FREE CONSTRUCTIONS OF GROUPS BY ROOT CLASSES

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Let  $\mathcal{C}$  be a class of groups. A group  $G$  is called *residually a  $\mathcal{C}$ -group* if, for every non-trivial element  $g \in G$ , there exists a homomorphism  $\sigma$  of  $G$  onto a group from  $\mathcal{C}$  such that  $g\sigma \neq 1$ . We say that the class  $\mathcal{C}$  is *root* if it contains at least one non-trivial group and is closed under taking subgroups, extensions, and Cartesian products of the form  $\prod_{y \in Y} X_y$ , where  $X, Y \in \mathcal{C}$  and  $X_y$  is an isomorphic copy of  $X$  for each  $y \in Y$ .

The use of the concept of root class turns out to be quite productive in the study of the approximability of free constructions of groups: generalized free products, HNN-extensions, tree products, fundamental groups of graphs of groups. One of the main methods for exploring the residual properties of such constructions is the so-called “filtration approach” of G. Baumslag. Initially, it was proposed to study the residual finiteness of generalized free products of two groups, and then it was extended to other free constructions and adapted to study the property of residual  $p$ -finiteness. It turns out that in most cases this method can be used to explore the approximability by an arbitrary root class of groups. This makes it possible to significantly increase the number of results obtained, as well as to apply them to the study of the approximability by some non-root classes of groups.

The talk describes the use of the filtration approach to study the approximability of free constructions of groups by root classes and presents the latest results obtained using this method.

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