5. Выполненное исследование базируется на развитом алгебро-грамматическом аппарате, включающем логические средства и метапрограммы выводимости схем алгоритмов и программ, принадлежащих классам, ассоциированным с актуальными предметными областями. Особенность алгебро-грамматических средств представляет знание — гармоническое сочетание декларативных процедурных и трансформационных спецификаций, а также адекватность данных средств концепции объектно-ориентированного программирования.

На основе полученных результатов разработаны наукоемкая технология и ее инструментарий КЭС-система МУЛЬТИПРОЦЕССИСТ, на- лежшие применение при решении задач АСУ, САПР конструкторской и технологической подготовки производства, языковых процессов — ров транслирующего и интерпретирующего типа.

Литература


FINITARY LAMBDA CLONES

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There are known several algebraic structures aspiring to be an algebraic counterpart of the λ-calculus: λ-algebras, combinatorial models, λ-models (here and further we follow, on the whole, the terminology adopted in [1]). We present one more such a structure called a (finitary) λ-clone which has some attractive features from both, universal algebra and λ-calculus points of view. Namely, for any environment model of the λ-calculus, the range of the valuation mapping is a λ-clone, in fact, a so-called locally finite (l.f.) λ-clone. In addition, the very valuation mapping is nothing but a homomorphism onto this λ-clone and the corresponding λ-theory (of the model) is the kernel of this homomorphism. Equivalently, if (C, ', k, s) is a λ-algebra and X is a countable set of variables, then the polynomial algebra C[X] can be equipped with the structure of a λ-clone and proves the l.f. λ-clone generated by C (not X!) with certain defining relations.
At the same time, the class of all \( \lambda \)-clones is a variety of algebras contained in the class of \( \lambda \)-models and, besides, the last one, considered as a category, is equivalent to the full subcategory of \( \lambda \)-clones consisting of \( l.f. \) \( \lambda \)-clones.

These nice properties of \( \lambda \)-clones are provided by the point that variables, \( \lambda \)-quantifiers and substitutions are included directly into the signature of \( \lambda \)-clones. Roughly speaking, a \( \lambda \)-clone is an abstract clone in the sense of universal algebra equipped with operations of application and \( \lambda \)-quantifications.

**DEFINITION 1** [2]. Let \( L = (x_i, s_i, ', \lambda_i)_{i<\omega} \) be a signature of operation symbols where all \( x_i \) are constants, all \( s_i \) and ' are binary and all \( \lambda_i \) are unary. A (finitary) \( \lambda \)-clone is defined to be an \( L \)-algebra \( W \) s.t. the following identities hold for each \( i, j, k < \omega \) with \( i \neq j \neq k \) (\( u, w \) range over the underlying set \( W \), \( v \) stands for \( s_j(x_k, w) \)):

\[
\begin{align*}
(S) & \quad s_i(x_i, w) = w, \quad s_i(w, x_i) = w, \quad s_j(w, x_i) = x_i, \quad s_i(u, v) = v, \\
& \quad s_i(u, w_1, w_2) = s_i(u, w_1), s_i(u, w_2); \\
(SQ) & \quad s_i(w, \lambda_i u) = \lambda_i u, \quad s_i(v, \lambda_i u) = \lambda_i s_i(v, u); \\
(a) & \quad \lambda_i v = \lambda_i s_i(x_i, v); \quad (\beta) \quad (\lambda_i w), u = s_i(u, w).
\end{align*}
\]

Here \( (S) \) - identities correspond to intuitive understanding of \( s_i(u, w) \) as the result of substituting of \( u \) into \( w \) for \( x_i \), \( (S) \) - identities regulate interaction between substitutions and \( \lambda \)-quantification, \( (a) \), \( (\beta) \) provide \( (a), (\beta) \) - conversion resp.

Given a \( \lambda \)-clone \( W \), for each element \( w \in W \) we introduce its dimension set, \( \Delta w := \{ i < \omega : s_i(u, w) \neq w \text{ for some } u \neq x_i \} \), and call \( w \) finitary if \( |\Delta w| < \omega \) and closed if \( \Delta w = \emptyset \). Further, we define \( W_{\text{fin}} := \{ w : |\Delta w| < \omega \} \) and \( W_\emptyset := \{ w : \Delta w = \emptyset \} \). \( W_{\text{fin}} \) proves a subalgebra of \( W \), \( W_\emptyset \), and \( W \) is called locally finite dimensional (l.f.) if \( \emptyset = W_{\text{fin}} \).

**DEFINITION 2.** An environment domain is defined to be a pentuple \( \mathfrak{g} = (D, V, F, \phi, \psi) \) with \( D \) a set, \( V \) a set of operations \( D^\omega \rightarrow D, F \) a set of operations \( D \rightarrow D, \phi \) a mapping \( D \rightarrow F \) and \( \psi \) a mapping \( F \rightarrow D \) s.t. the following conditions are fulfilled (we use the symbol \( \Lambda \) for the ordinary set theoretical (meta)}
lambda abstraction): 

(E1) \( \pi_i := (\Lambda \rho \in D^\omega . \rho i) \in V \) for all \( i < \omega \), \( \bar{d} := (\Lambda \rho \in D^\omega . d) \in V \) for all \( d \in D \);  

(E2) if \( u, v \in V \) then \( (\Lambda \rho \in D^\omega . v([u/i]_\rho)) \in V \) and \( (\Lambda \rho \in D^\omega . \Phi(u)(v_\rho)) \in V \);  

(E3) if \( v \in V \) then, for any fixed \( i < \omega, \rho \in D^\omega \), \((\Lambda d \in D. v([i/d]_\rho)) \in F \).

An environment domain is said to be an environment model if

(E4) \( \Phi \mathcal{W} = \mathcal{F} \) for all \( f \in F \).

CONSTRUCTION.

(i) With any environment domain \( \mathcal{E} \) as above there is correlated the L-algebra \( \mathcal{V}_\mathcal{G} = (V, \pi_i, s_i, ', \lambda_i) \) with operations defined according to the items (E1, 2, 3) of the definition 2.

(ii) With any L-algebra \( \mathcal{W} \) there is correlated a domain pentuple \( \mathcal{S}_\mathcal{W} = (D, V, F, \phi, \psi) \) where \( D = W_\emptyset, V \) is the set of all operations on \( D \) determined by polynomials built from a countable set of variables and elements of \( D \) with the operation ', \( w = \lambda u \in W_\emptyset, w'u, F := \{ (w: w \in W_\emptyset) \}, \psi w := (\lambda, \lambda_j(x_i', x_j))'w \).

THEOREM 1. If \( \mathcal{E} \) is an environment model then \( \mathcal{V}_\mathcal{G} \) is a l.f. \( \lambda \)-clone; if \( \mathcal{W} \) is a \( \lambda \)-clone then \( \mathcal{S}_\mathcal{W} \) is an environment model; finally, \( \mathcal{S}_\mathcal{W} \mathcal{E} \cong \mathcal{E} \) and \( \mathcal{V}_\mathcal{W} \mathcal{G} \cong \mathcal{W} \mathcal{F} \).

THEOREM 2. A \( \lambda \)-theory is defined to be a couple \((C, T)\) with \( C \) a set of constants and \( T \) a subset of \( \Lambda(C) \times \Lambda(C) \) closed under the ordinary \( \lambda \)-calculus deducibility. If \( \mathcal{T} = (C, T) \) is a \( \lambda \)-theory then \( \mathcal{W}_\mathcal{T} = \Lambda(C)/T \) is a l.f. \( \lambda \)-clone; if \( \mathcal{W} \) is a \( \lambda \)-clone and \( \mu \) is the homomorphism \( \Lambda(W_\emptyset) \rightarrow \mathcal{W} \) naturally extending the identity inclusion \( W_\emptyset \hookrightarrow W \), then \( \mathcal{F}_\mathcal{W} = (W_\emptyset, \ker \mu) \) is a \( \lambda \)-theory; finally, \( \mathcal{F}_\mathcal{W} \mathcal{T} \cong \mathcal{T} \) and \( \mathcal{W}_\mathcal{F} \mathcal{W} \cong \mathcal{W}_\mathcal{F} \).

CONJECTURE. The variety generated by the class of all l.f. \( \lambda \)-clones coincides with the class of all \( \lambda \)-clones.

IN PROSPECT. The notion of a finitary \( \lambda \)-clone is somewhat unnatural as finitary operations acting on closed elements can not reach elements with infinite dimension sets. In this con-
text, a more natural structure is a $\lambda$-clone with infinitary, 
$(1+\varpi)$-ry, applications and infinitary, $\overline{\varpi}$-ry, $\lambda$-quantifiers for 
all, finite and countable, sequences $\overline{\varpi} \in \omega^\omega$. In this way we ob-
tain an algebraic version of an infinitary $\lambda$-calculus but this 
is another story.

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ON CODING OF HEREDITARILY-FINITE SETS, POLYNOMIAL-TIME COMPUTA-
BILITY AND $\Delta$-EXPRESSIBILITY

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This paper is devoted to computability and definability 
in terms of bounded (i.e.,$\Delta$-) set theoretic language (cf. re-
ferences below).

A coding (or numbering; cf. the general theory in [3]) of 
the universe of hereditarily-finite sets HF is any surjection 
$\Theta: A^* \rightarrow HF$ from the set of all finite strings over some finite 
alphabet $A$. Let $P_\Theta$ denote the class of operations $P:
HF \rightarrow HF$ such that $P \Theta = \Theta f$ for some polynomial-time computable 
(or shortly, $P$-) function $f: A^* \rightarrow A^*$. For any two codings 
$\Theta: A^* \rightarrow HF$, $\eta: B^* \rightarrow HF$ and $P$-function $f: A^* \rightarrow B^*$ the $P$-redu-
cibility $\overset{f}{\Theta} = \Theta f$ is denoted also as $\Theta P^f \eta$ or $\Theta \leq_P \eta$. 
$P$-equivalence $\Theta P^f \eta$ means $\Theta \leq_P \eta$ & $\eta \leq_P \Theta$ and implies 
$P \Theta = P$. If cardinalities of $A$ and $B$ are $\geq 2$ then any 
$\Theta: A^* \rightarrow HF$ is $P$-equivalent to some $\Theta B^* \rightarrow HF$ (via arbitrary 
two-sided $P$-bijections $f:A^* \rightarrow B^*$). Hence, we will usually consi-
der codings over the same $A$. Any $\Theta$ is called $P$-coding if (1) 
the predicate "HF $\models \Theta(a) \in \Theta(b)"$ is $P$-decidable on any, $a,b \in 
\in A^*$ and (2) two $P$-computable mappings $a \rightarrow a_1, \ldots, a_k$ and $a_1,\ldots